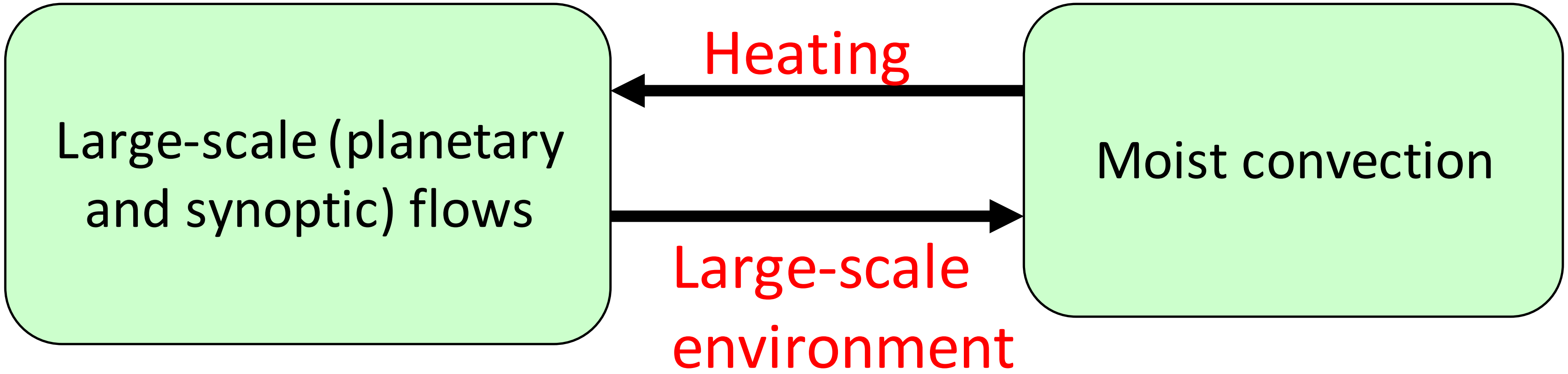
Applications of linear response functions in moist and jet dynamics

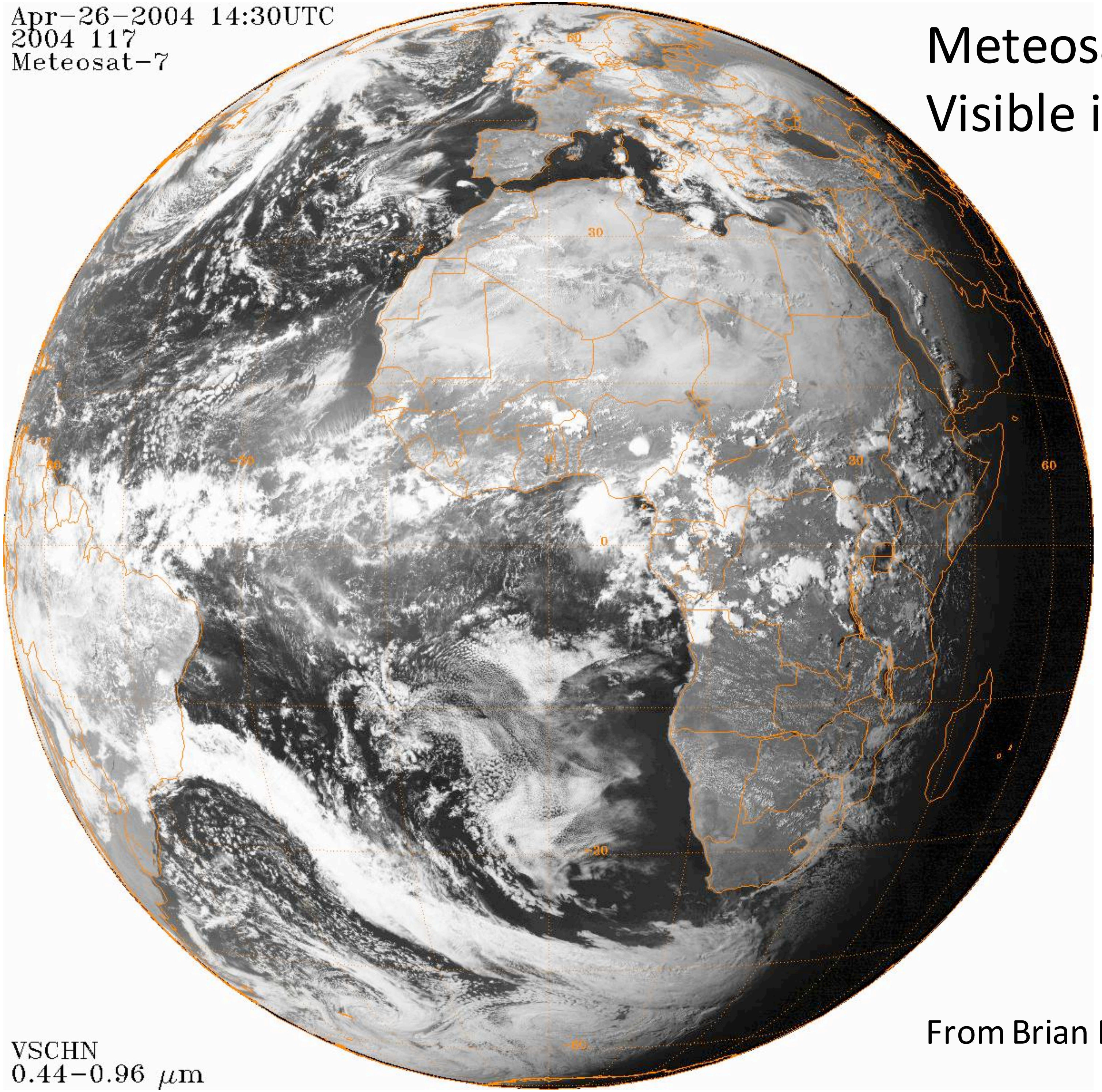
Zhiming Kuang

Dynamics of moist atmospheres

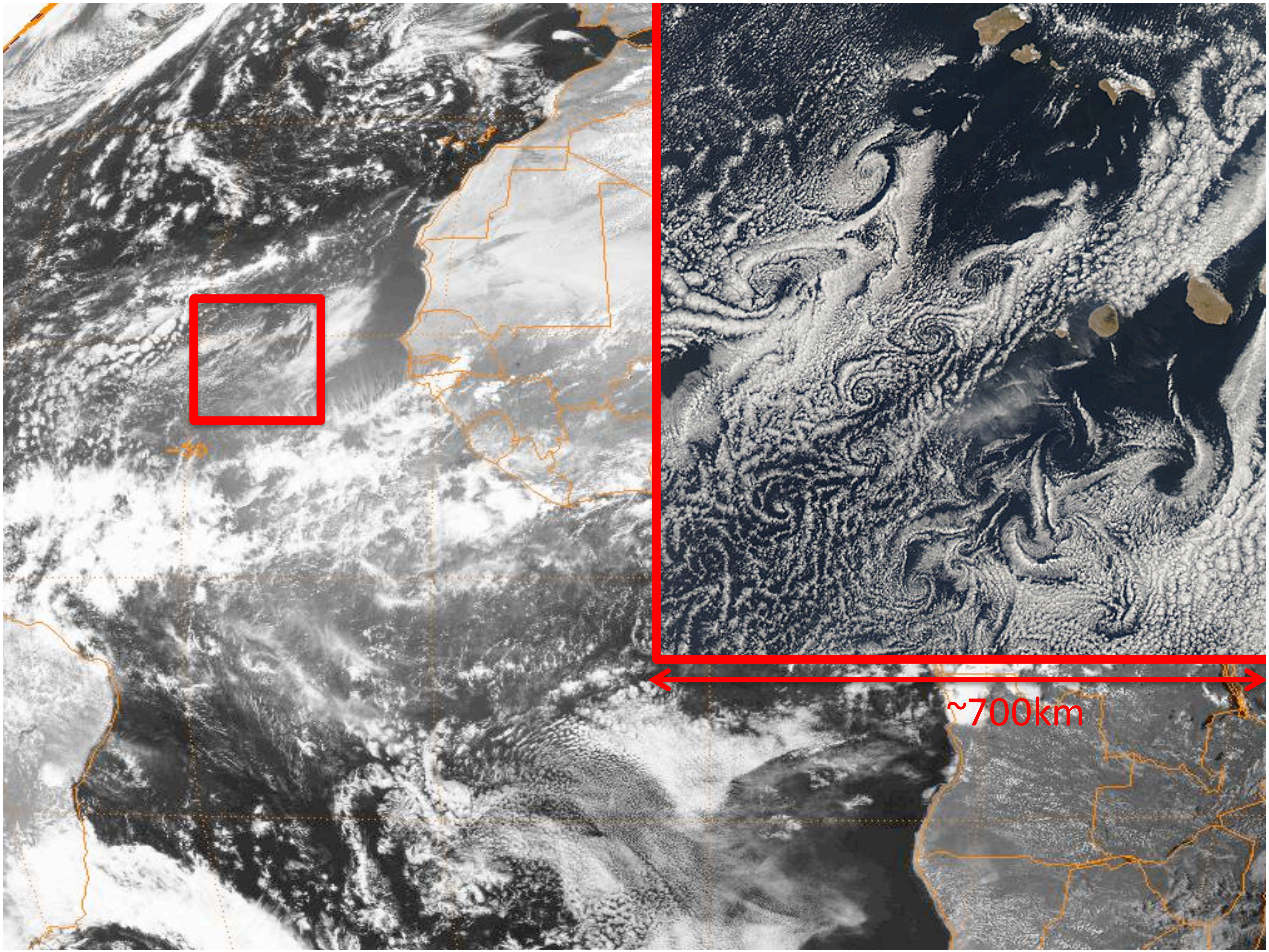


The strong coupling between convection and large-scale circulations is central to the dynamics of moist atmospheres

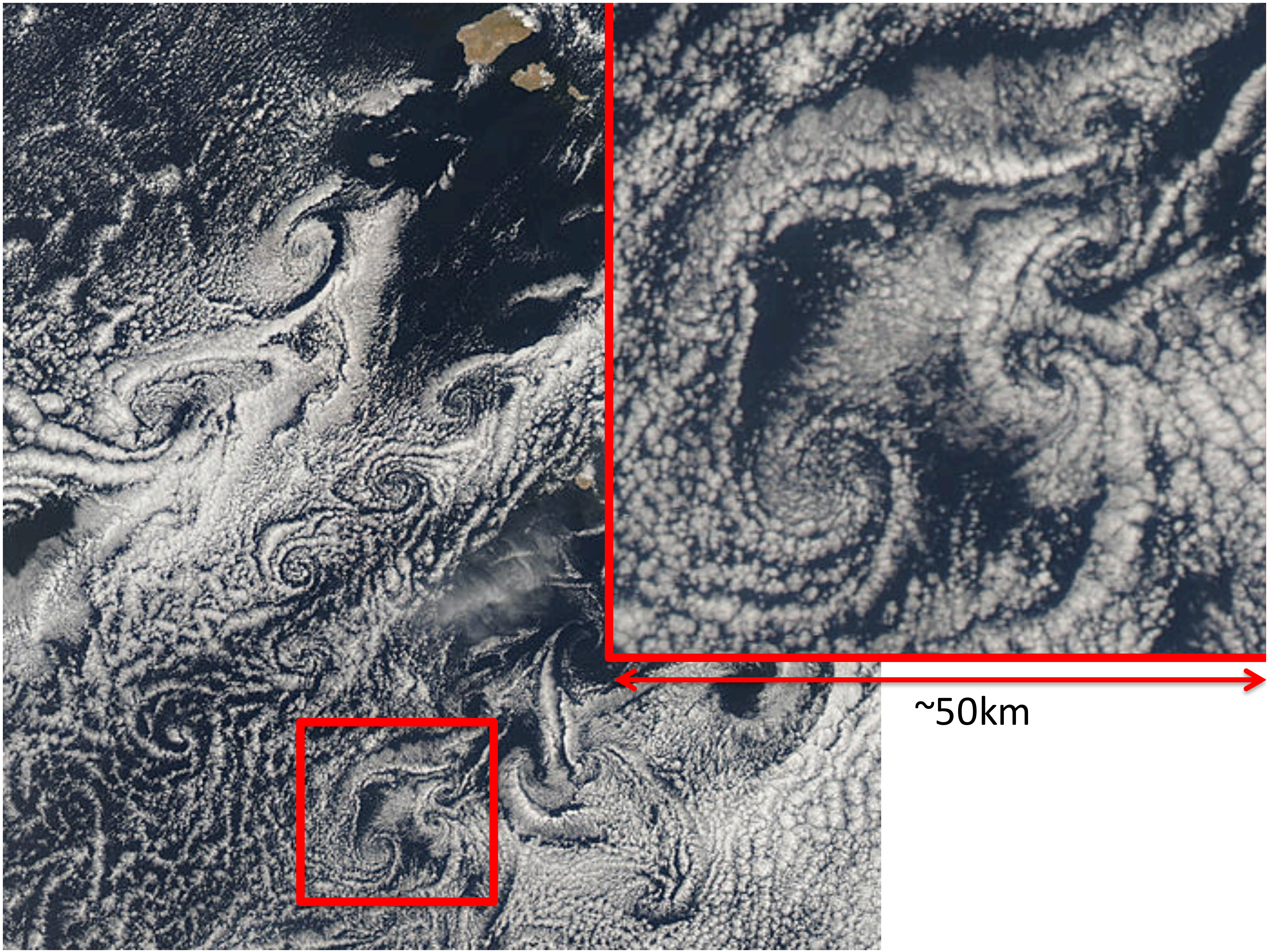
Meteosat-7 Visible image



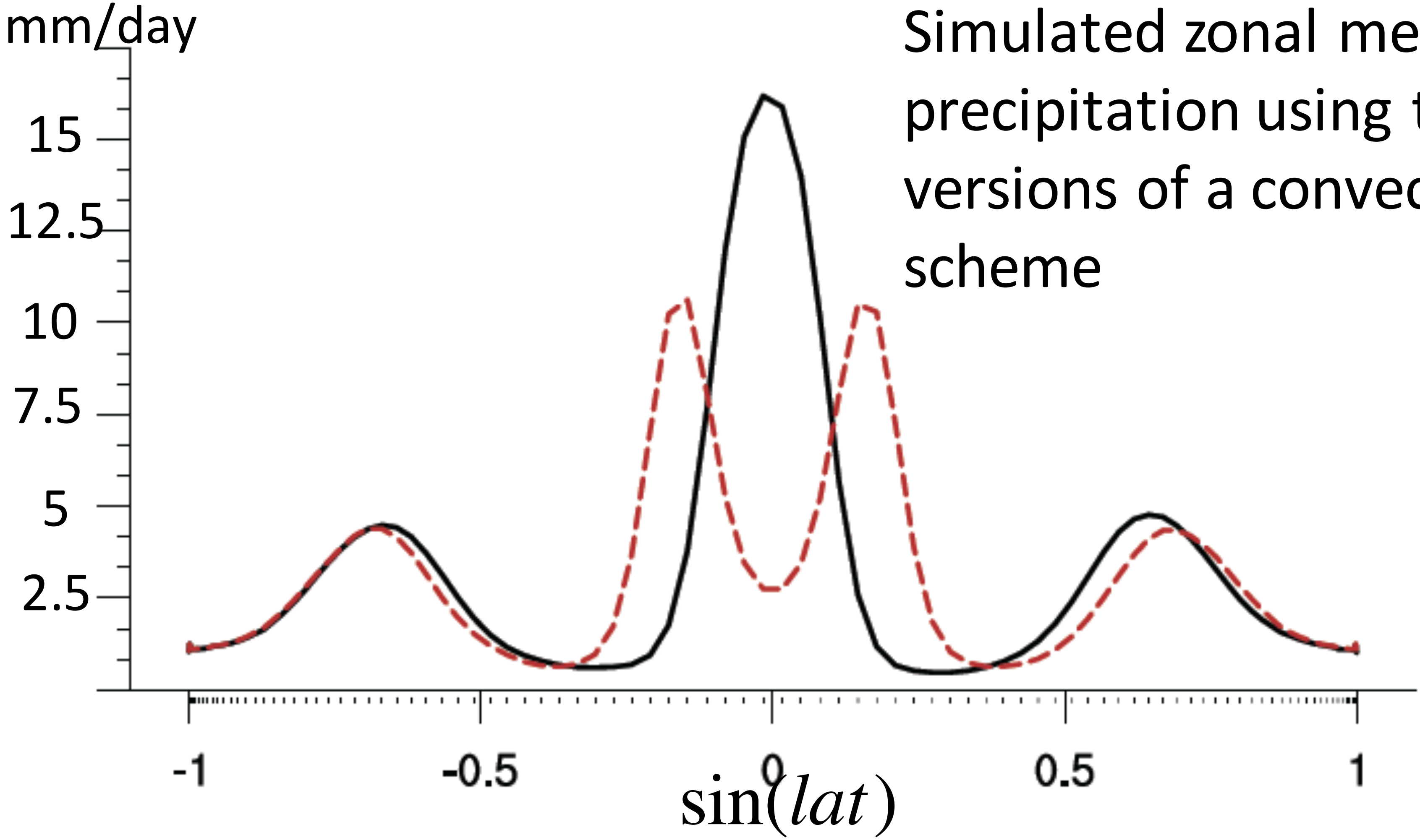
From Brian Mapes



~700km


~50km

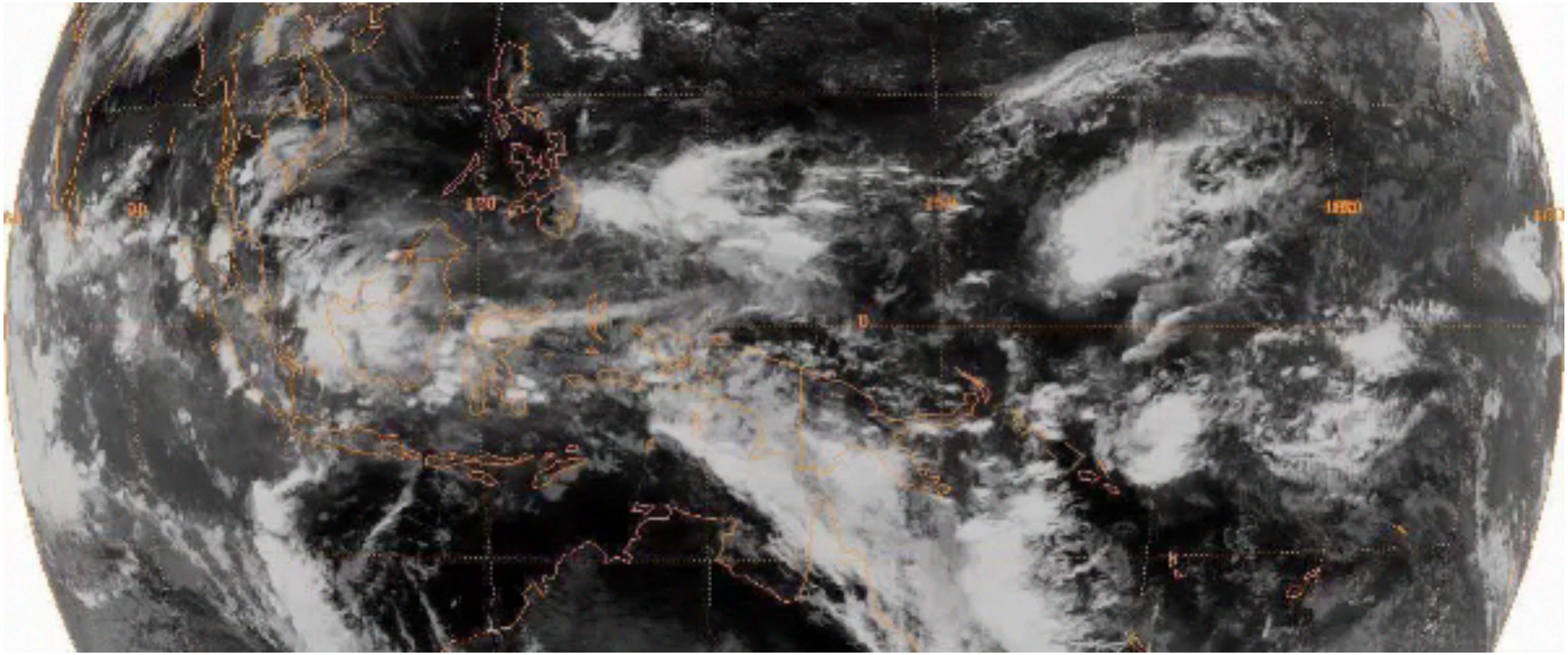
Aquaplanet simulations that differ only in their representations of convection



Simulated zonal mean precipitation using two versions of a convection scheme

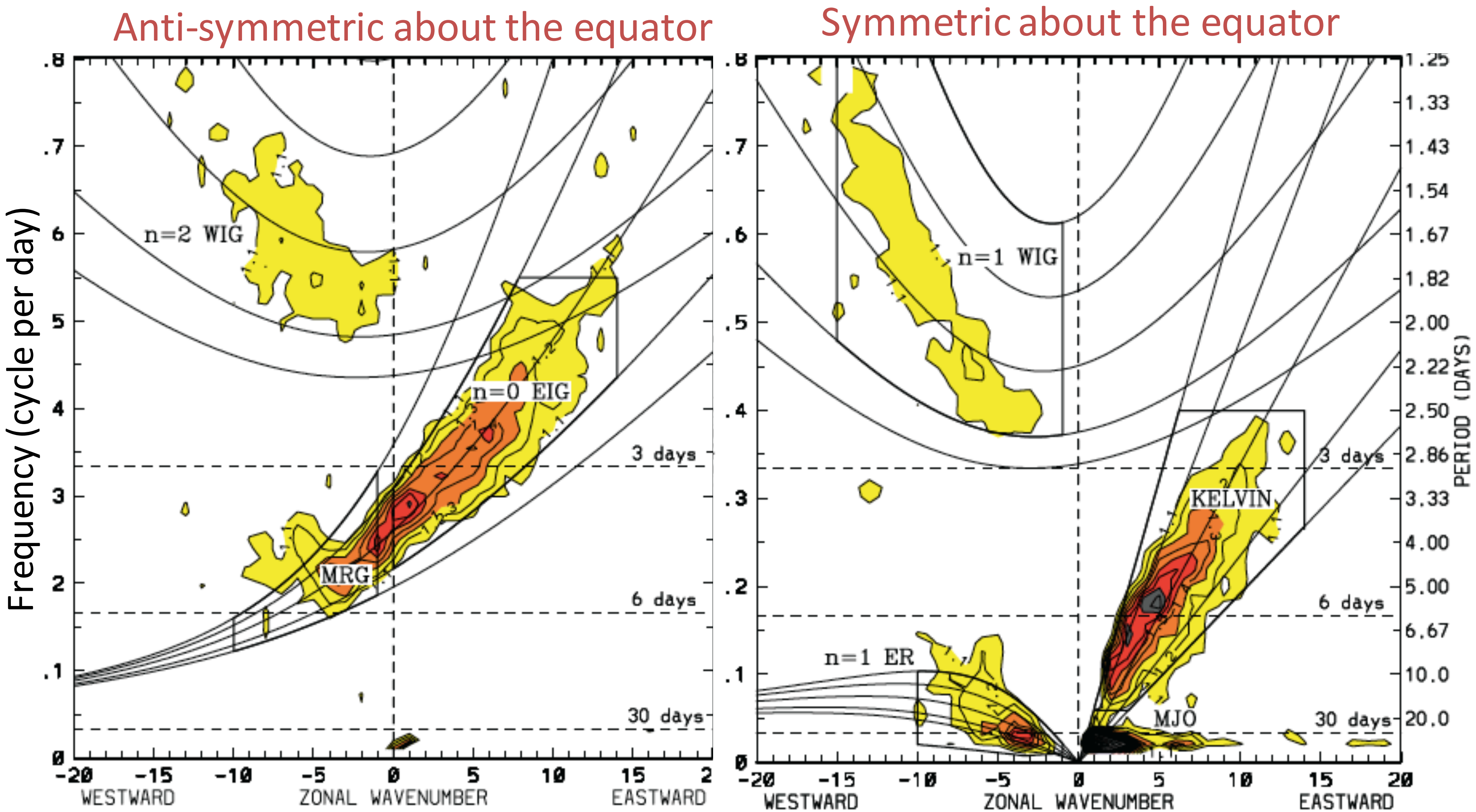
Courtesy of Bjorn Stevens, Following Hess et al., J. Atmos. Sci., 1993

Tropical transients



Space-time spectra (averaged over 15N-15S, 20 years)

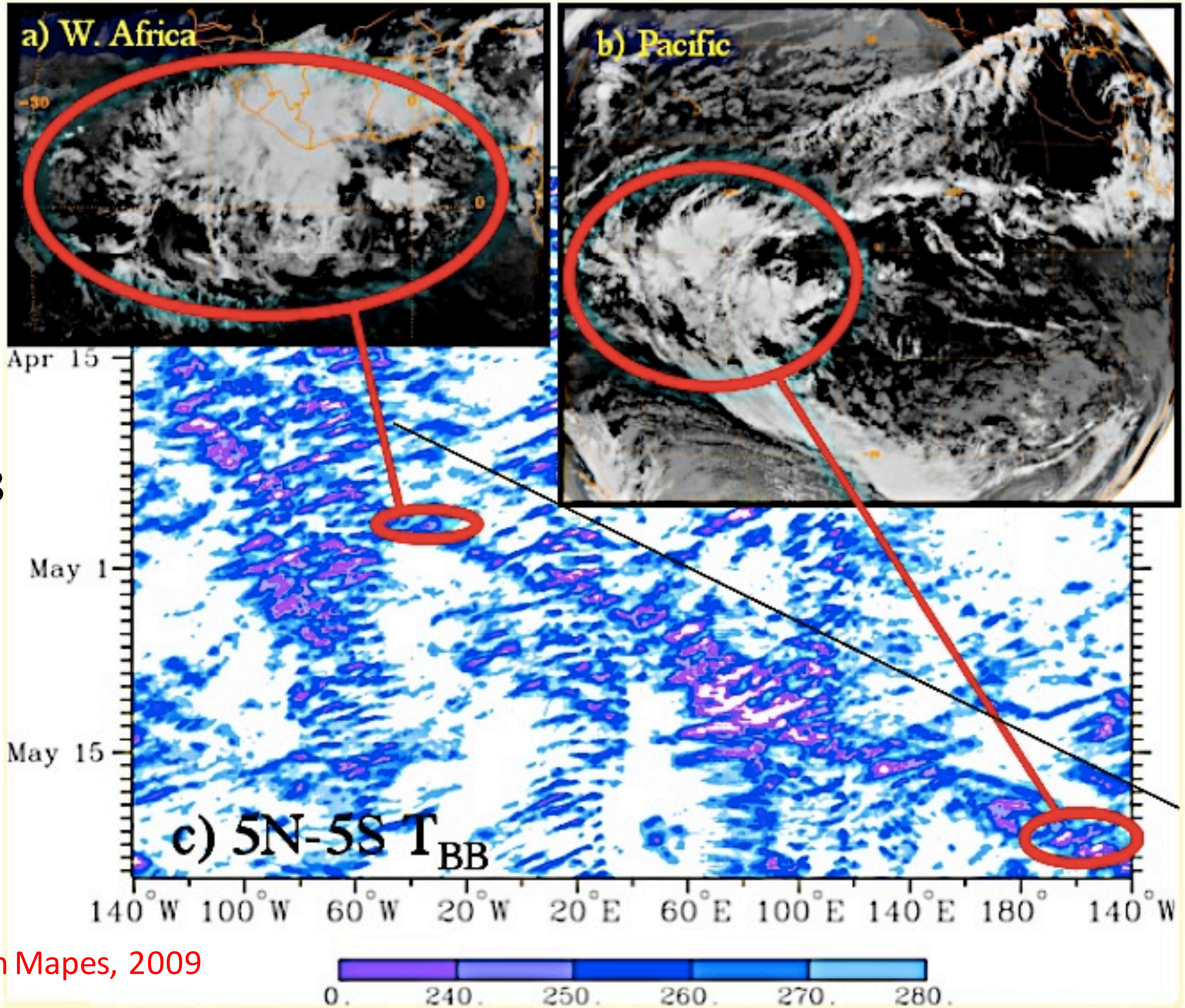
(with background red noise removed)



Overlaid are dispersion curves of linear equatorial shallow water modes of Matsuno 1966

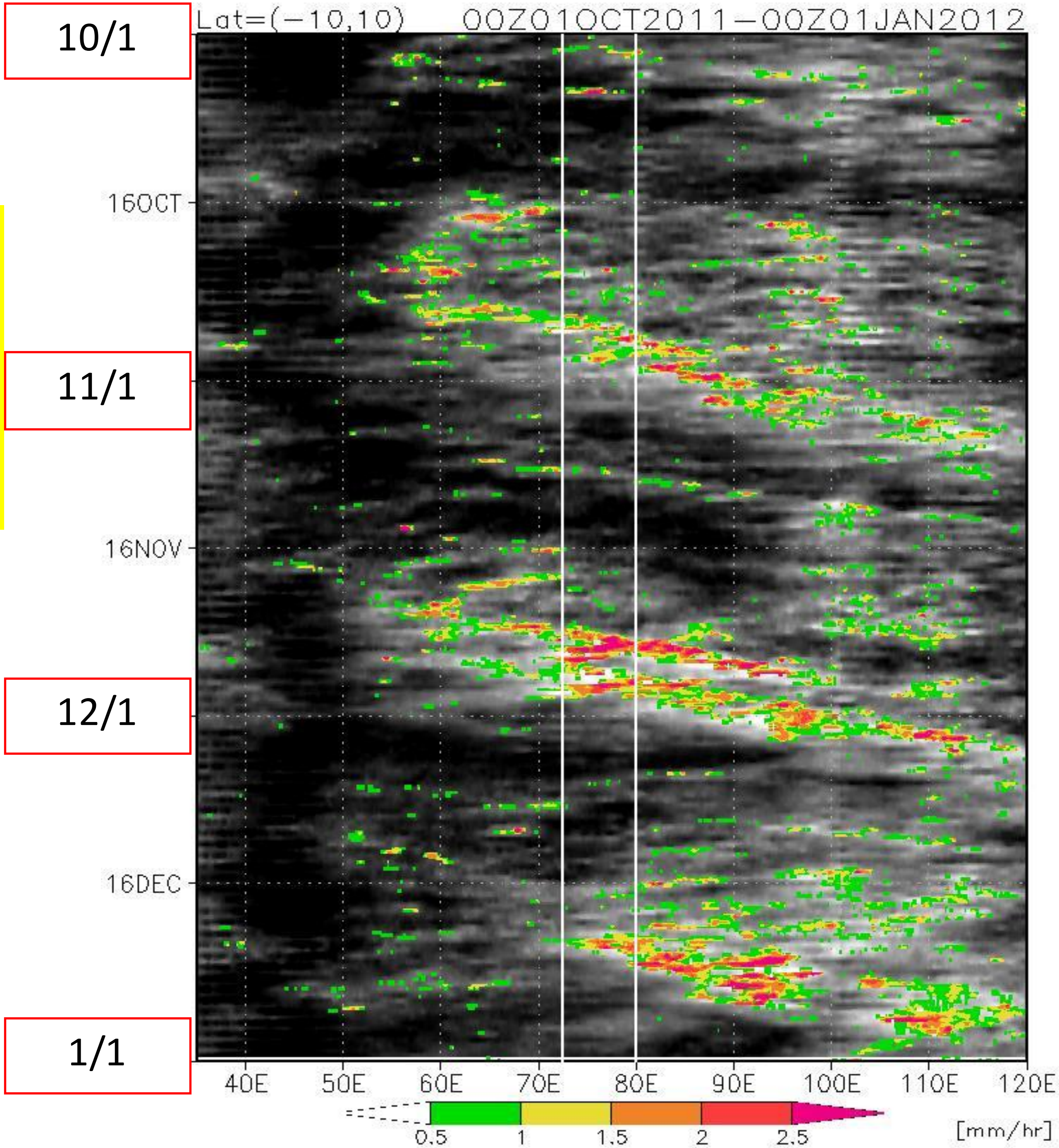
After Wheeler and Kiladis, J. Atmos. Sci., 1999

1998



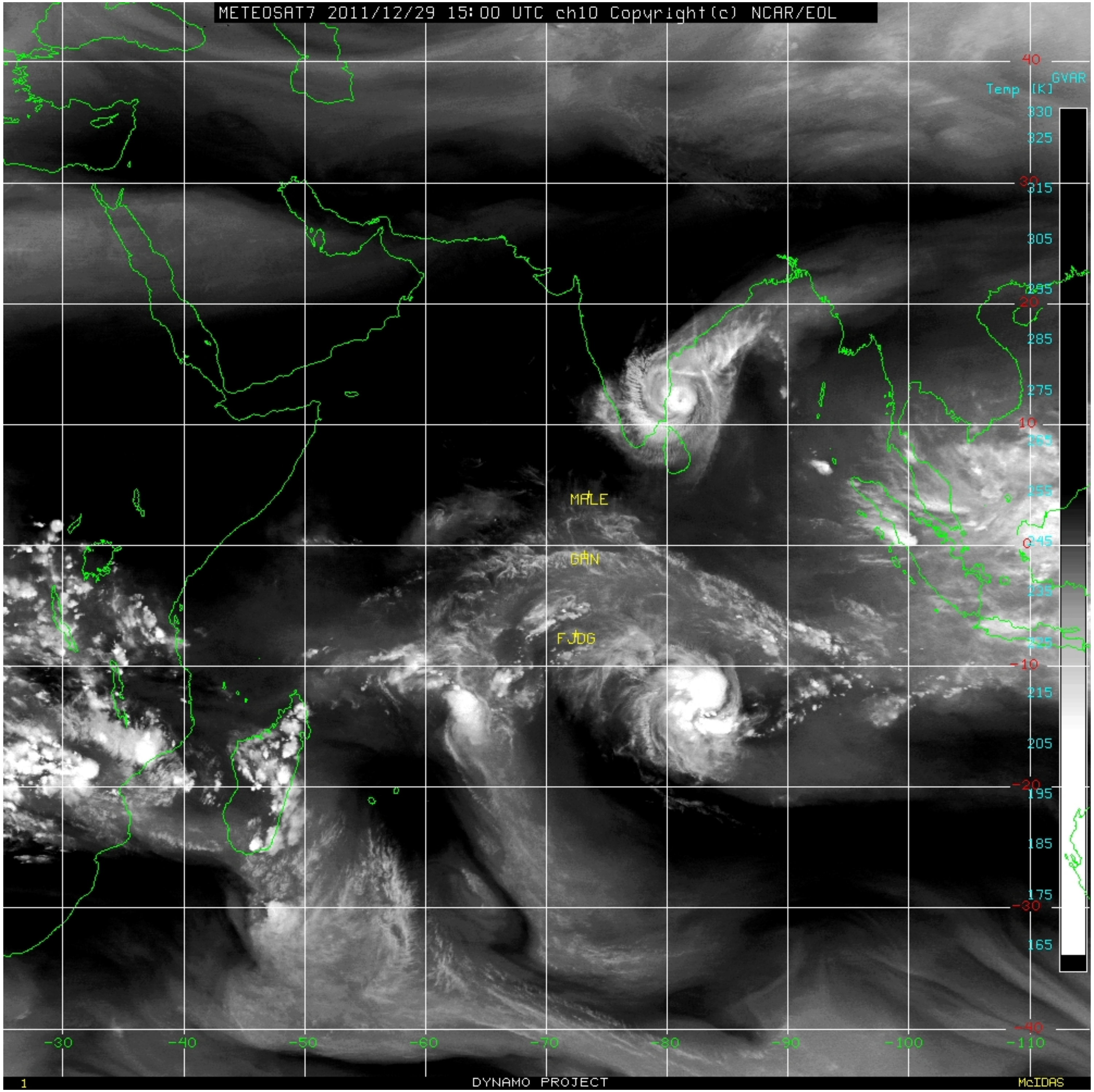
From Mapes, 2009

Examples from the 2011-2012 DYNAMO field campaign



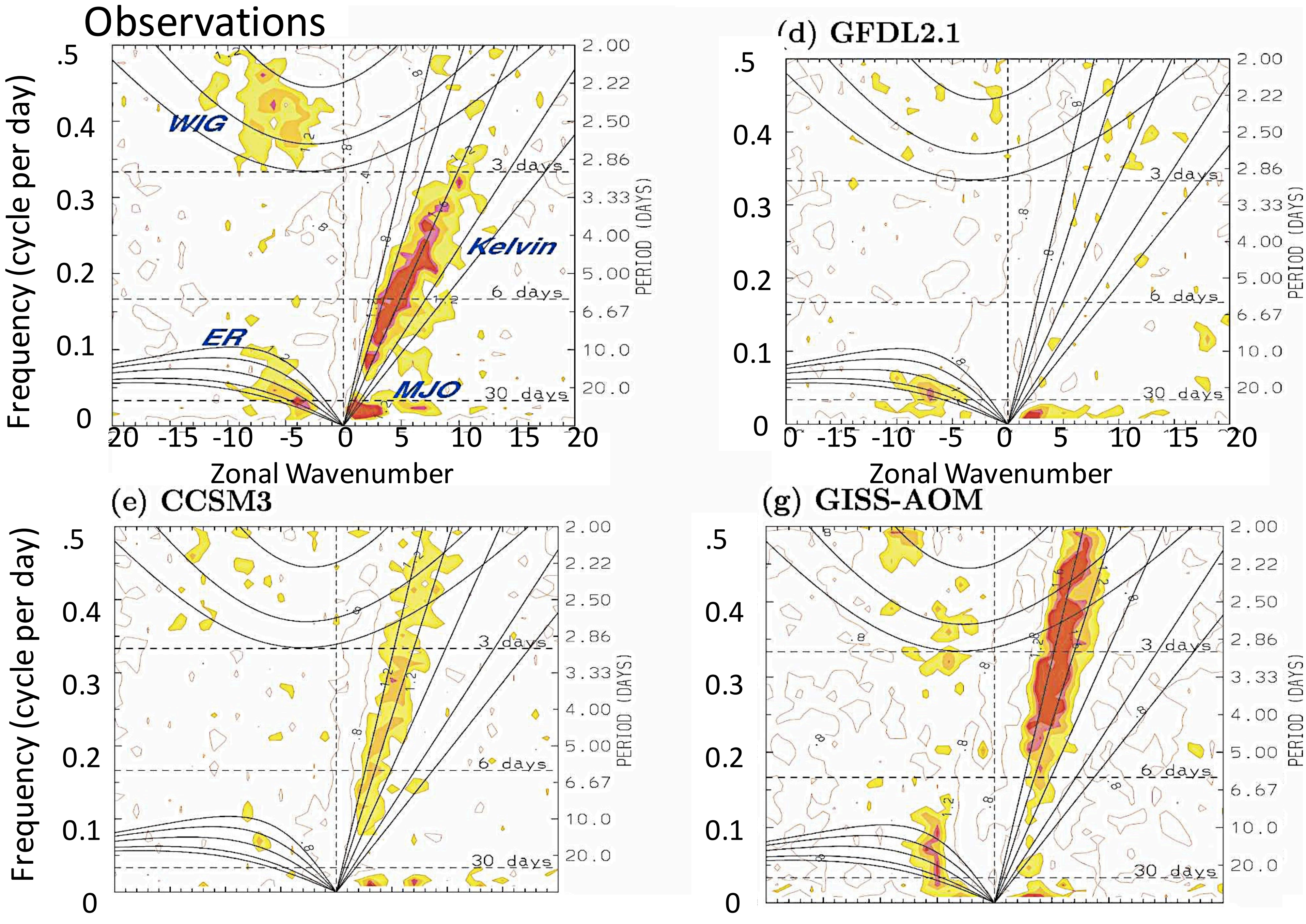
Courtesy of Kunio Yoneyama and Chidong Zhang

Dec. 29, 2011
METEOSAT7
Ch10
Water vapor

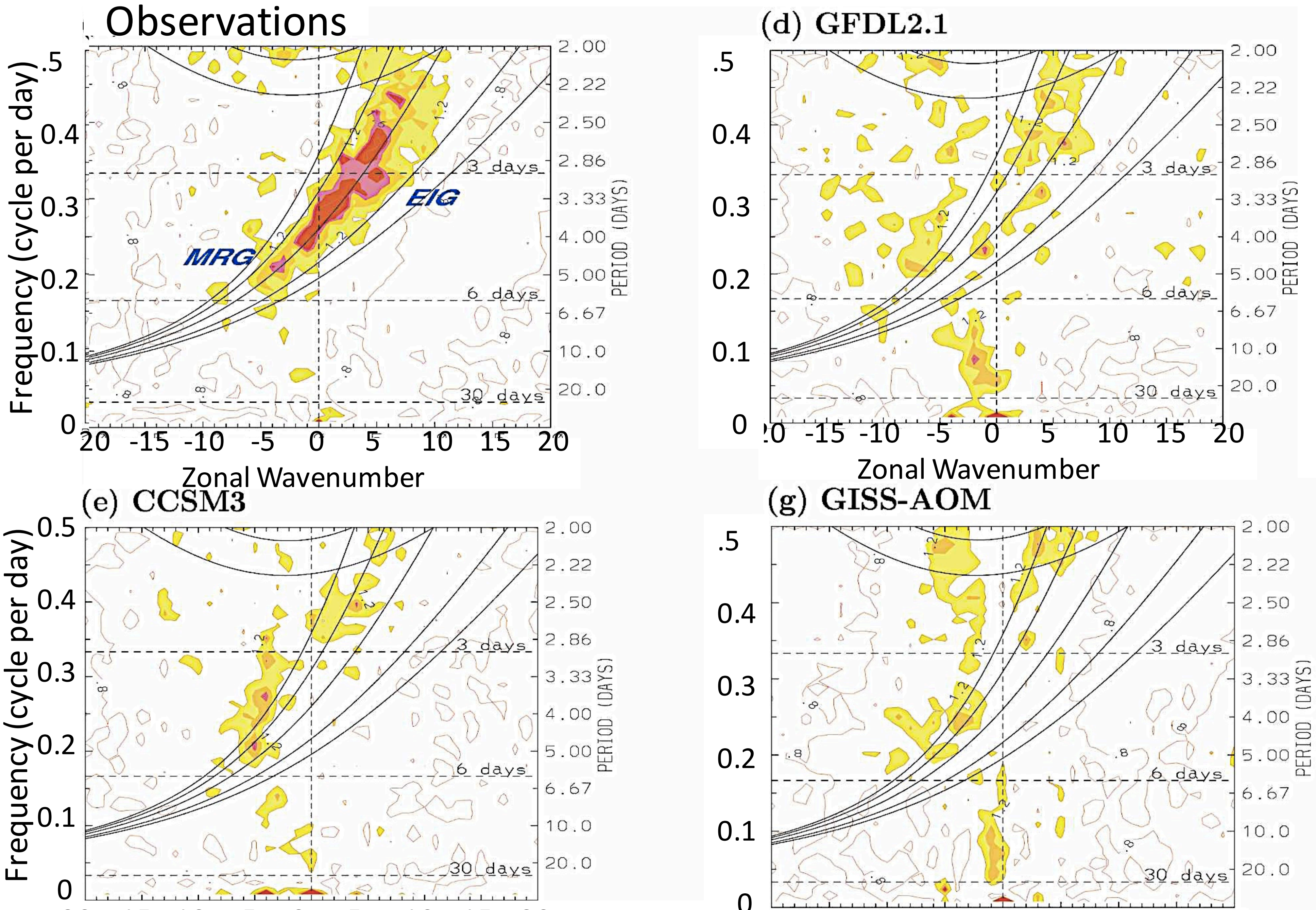


Why study convectively coupled tropical transients?

- Practical:
  - Tropical forecast, including monsoon, tropical cyclones etc. (e.g. Yasunari, 1979; Maloney and Hartmann, 2000)
  - ENSO (e.g. McPhaden, 1999)
  - Global medium range weather forecast (e.g. Ferranti et al., 1990)
- Theoretical:
  - Important examples of large-scale convective organization
  - A good starting point: quite well observed and convectively coupled waves appear linear

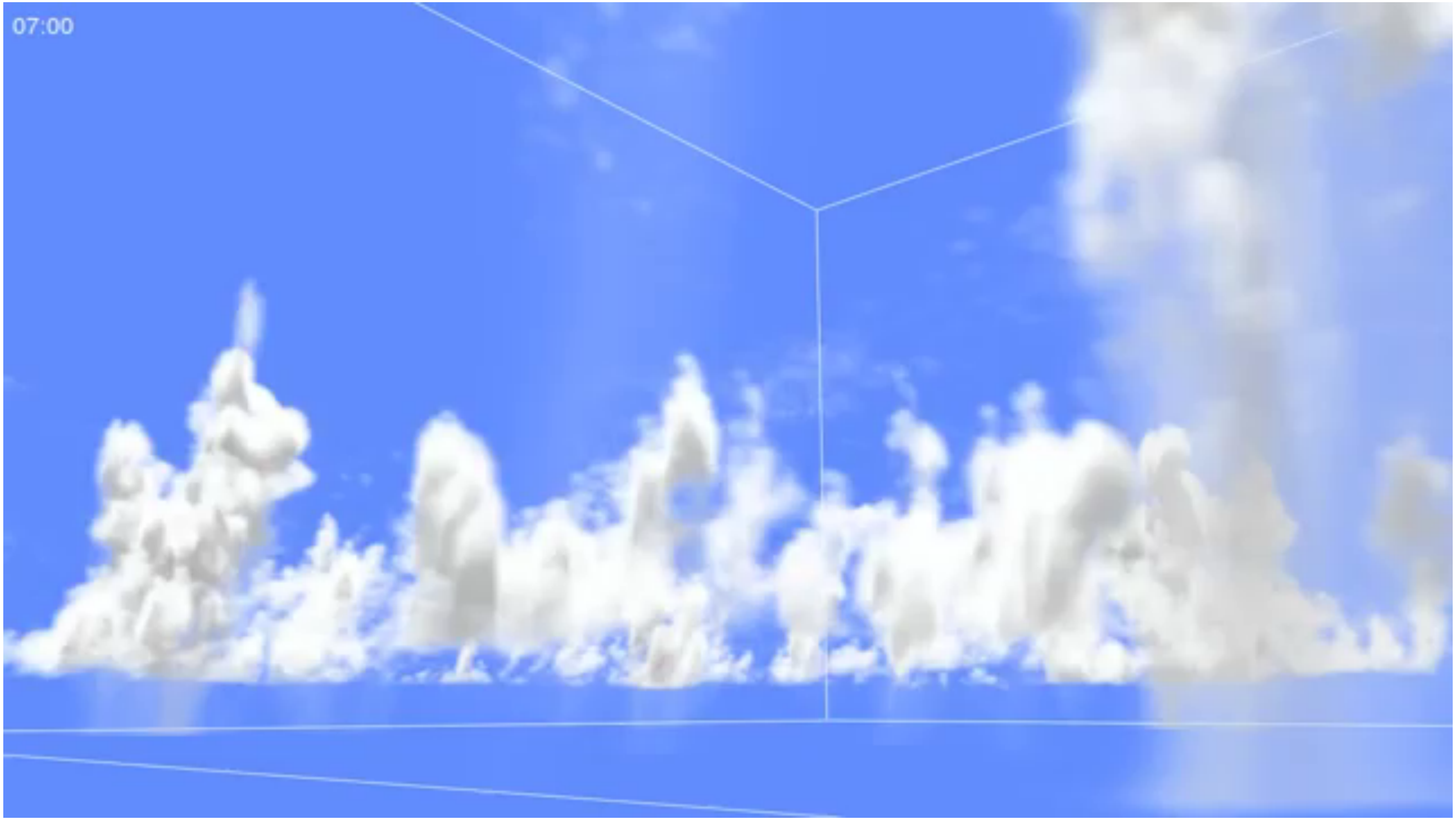


Lin et al., J. Climate, 2006, with IPCC AR4 models



Lin et al., J. Climate, 2006

Cloud-resolving models

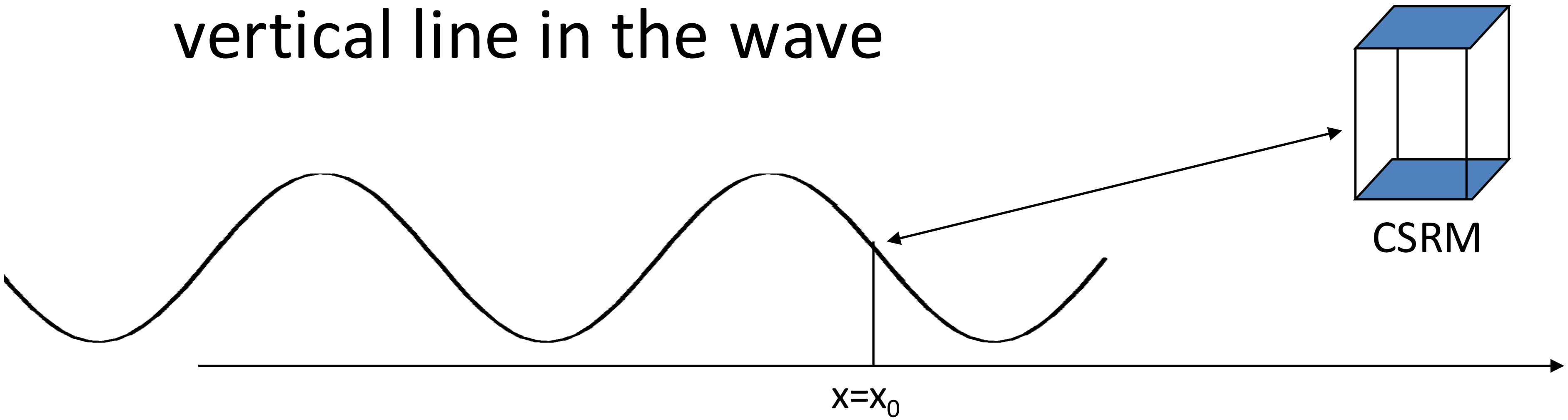


Credit: P. Siebesma, Delft U. Technology, Netherlands, computation done on a GPU

Simplify the problem

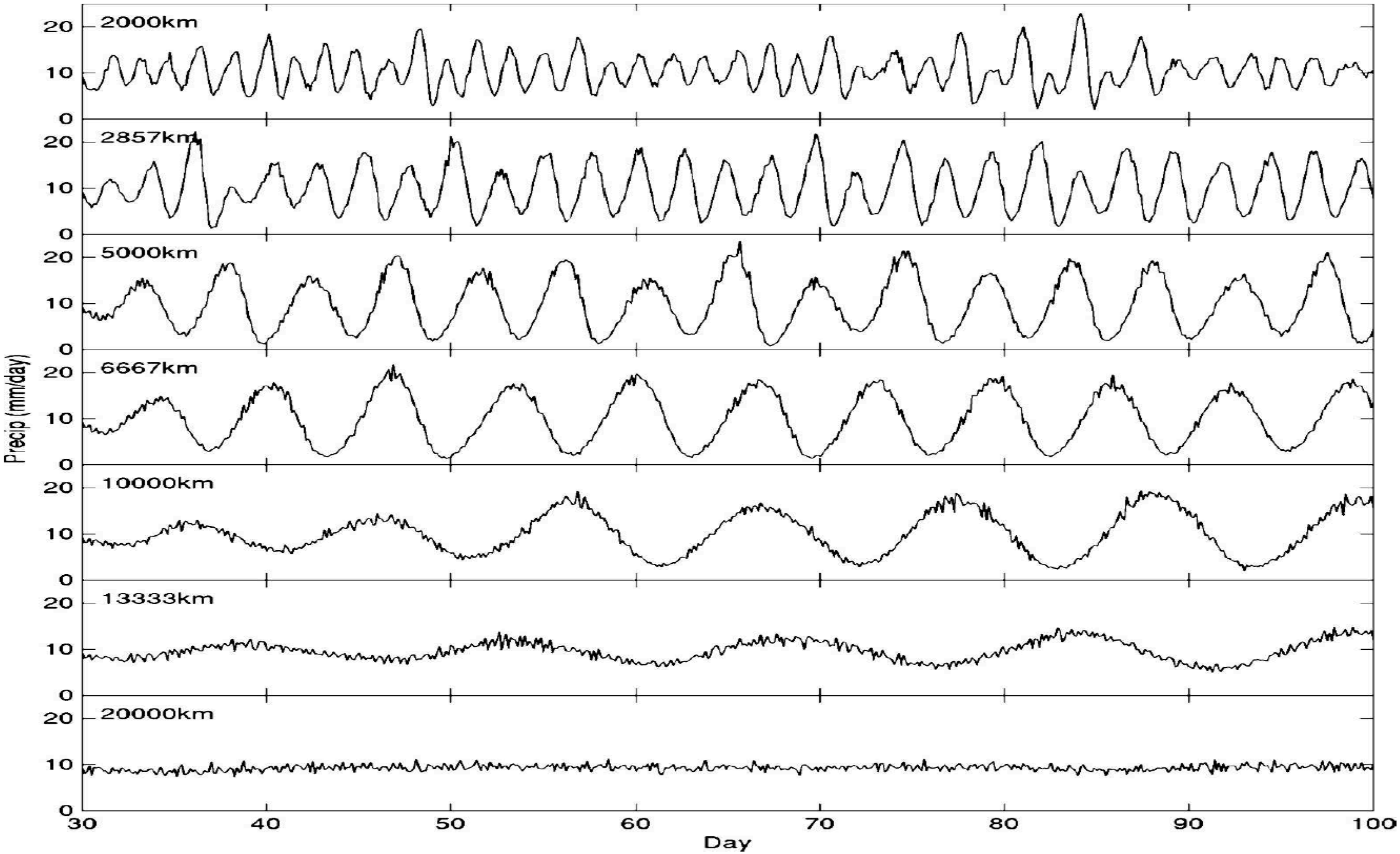
(both conceptually and computationally)

- Interaction between convection and two-dimensional (2D) linear gravity waves (also with no radiative or wind-induced surface flux feedbacks)
- Take advantage of the linearity! Treat one horizontal wavenumber at a time
- Use a cloud-resolving model to represent a vertical line in the wave



Development of convectively coupled waves

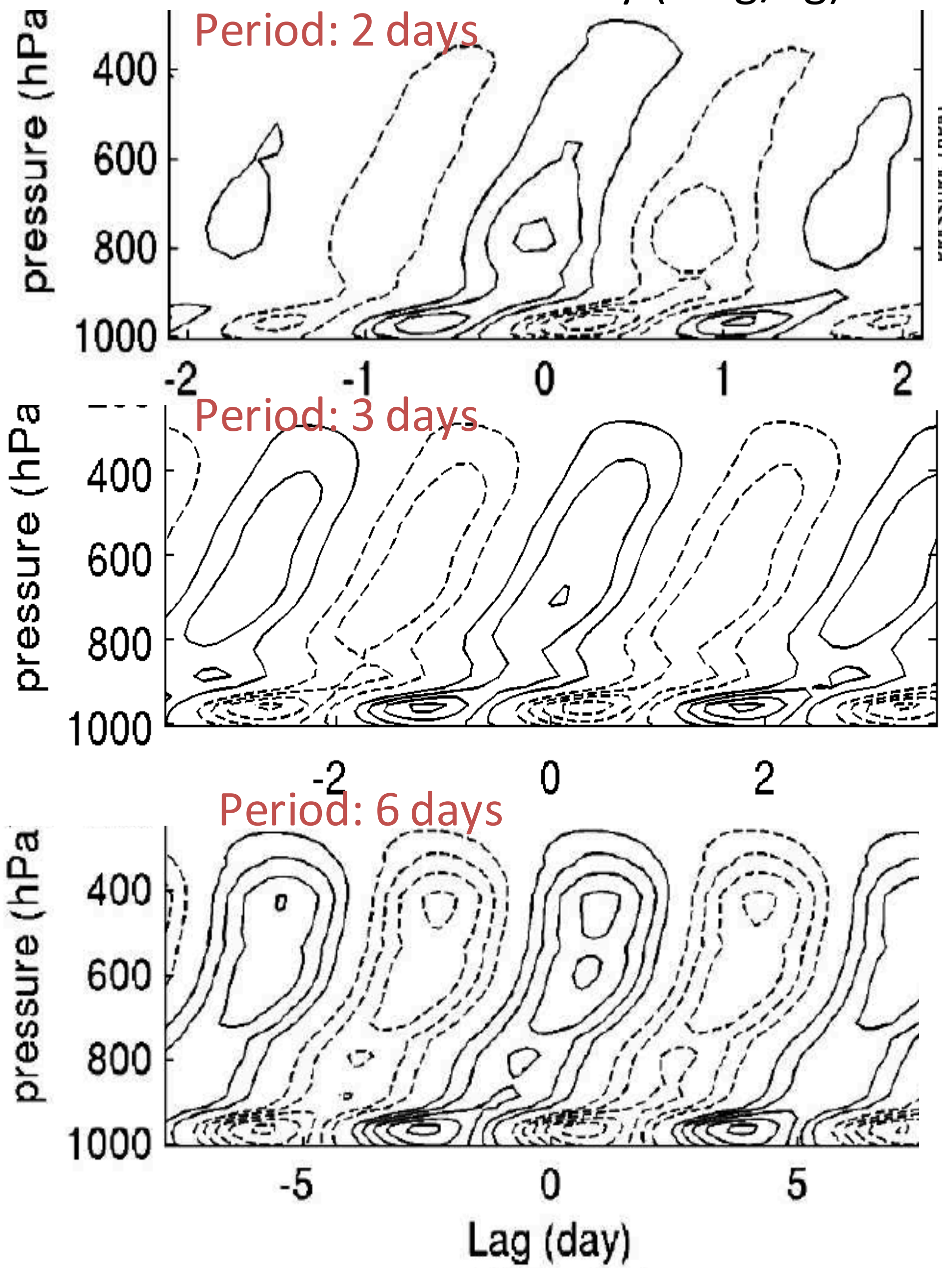
Cloud resolving model domain mean precipitation



Without coupling to gravity waves, the std of precip is 0.6mm/day

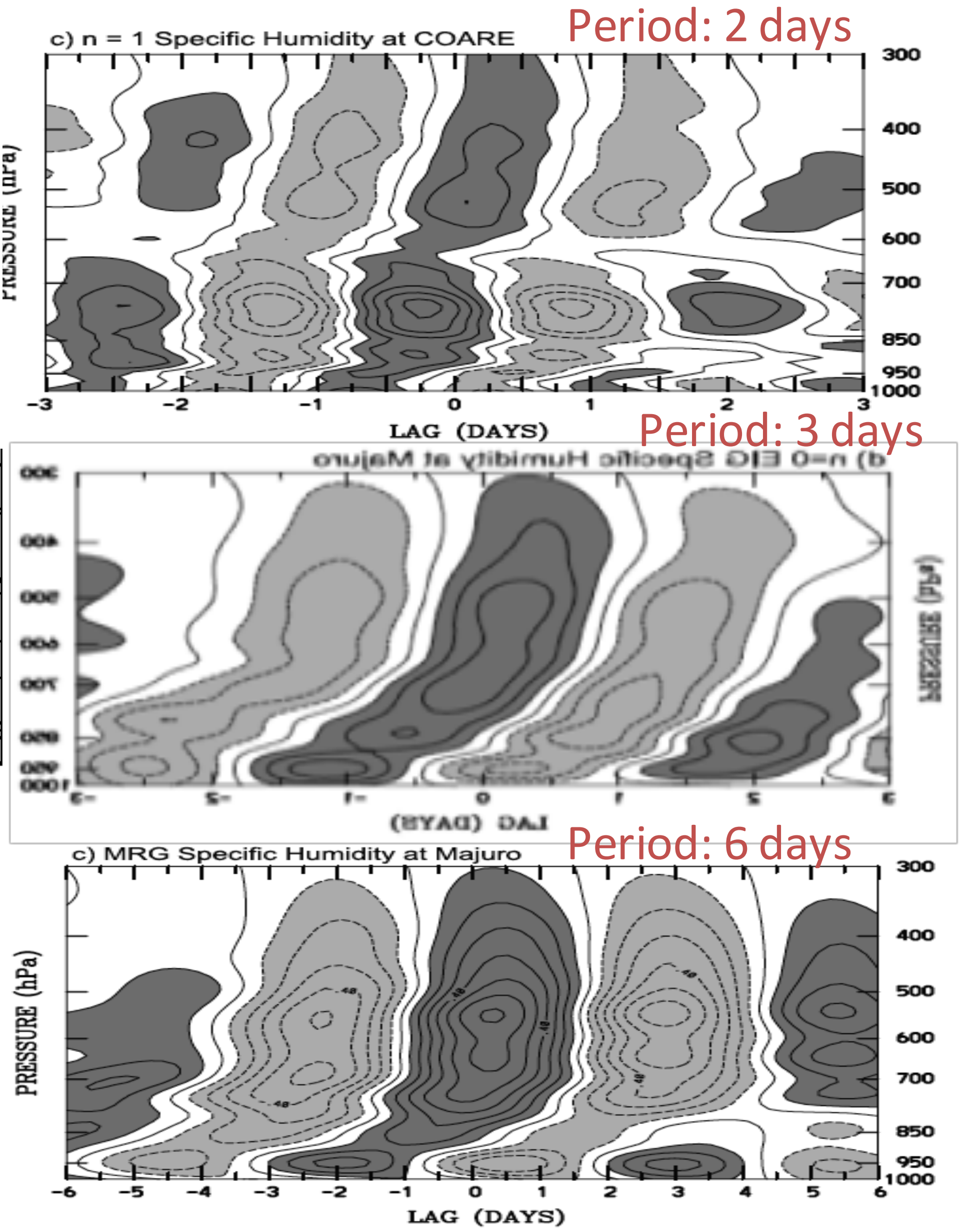
Simulation (Kuang, 2008)

Moisture anomaly (0.1g/kg)

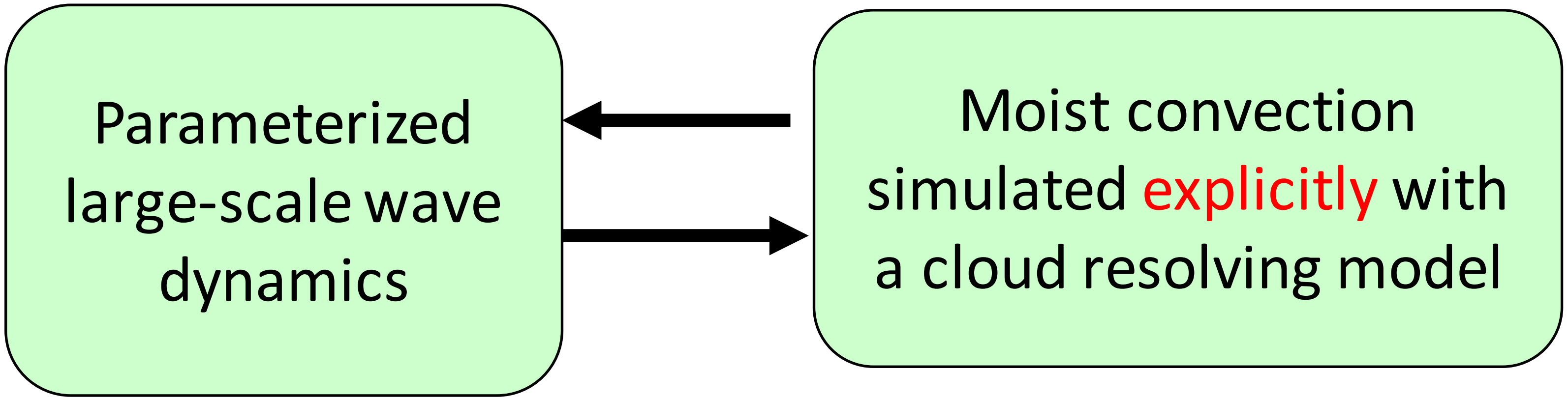


Observation

(Kiladis et al., 2009, Rev. Geophys.)



Looking for more clarity



Coupling with large-scale flow only cares about the macroscopic function (like the gas law), instead of the detailed form (like a description of all the molecules).

For convective coupled tropical waves, the macroscopic function of moist convection are captured by its linear response functions.

Consider a generic system

$$\frac{d\vec{x}}{dt} = \mathrm{S}(\vec{x}) + \vec{f}$$

$\vec{x}$ is the state vector that contains the variables that describe the system.

$\mathrm{S}(\vec{x})$ describes its evolution.

In the current example, $\mathrm{S}(\vec{x})$ is what is solved in the cloud-resolving model.

Now assume there is a reduced set of mean field variables $\vec{X}$ that full describe the system in a statistical sense

$$\frac{d\vec{X}}{dt} = \mathrm{R}\left(\vec{X}\right) + \vec{F}$$

i.e. statistics of $\vec{x}$ are in equilibrium with $\vec{X}$

In the current example, $\mathrm{R}\left(\vec{X}\right)$ describes a convective parameterization.

Further assume that $\mathrm{R}\left(\vec{X}\right)$ can be usefully linearized around a reference state $\vec{X}_0$ so that

$$\frac{d\vec{X}'}{dt} = \mathbf{M}\vec{X}' + \vec{F}'$$

$$\vec{X}' = \vec{X} - \vec{X}_0$$

$$\vec{F}' = \vec{F} - \vec{F}_0$$

$$0 = \mathrm{R}\left(\vec{X}_0\right) + \vec{F}_0$$

We will refer to $\mathbf{M}$ as the linear response function.

Note that $\mathbf{M}$ is a linearization of $\mathrm{R}\left(\vec{X}\right)$, not a linearization of $\mathrm{S}\left(\vec{x}\right)$.

Again $\mathbf{M}$ is a linearization of $\mathrm{R}\left(\vec{X}\right)$, not a linearization of $\mathrm{S}\left(\vec{x}\right)$, the original equations, nor is it an adjoint of the original model.

Past studies have tried to obtain $\mathbf{M}$ through the Fluctuation-Dissipation Theorem (FDT), which however suffer from the fact that the covariance matrix is often singular and the system is often non-normal (see Hassanzadeh and Kuang, 2016)

Linear response functions

$$\frac{d\vec{X}'}{dt} = \mathbf{M}\vec{X}'$$

- Define the (mean field) state vector $\vec{X}'$ to include profiles of large-scale T and q anomalies (horizontal winds can be included as well)
- This equation assumes that
  - Large-scale T, q completely describe the state of the atmosphere, i.e. moist convection is in statistical equilibrium with the T, q profiles. Reasonable for phenomena with periods of days or more.
  - Linearity holds for perturbations of relevant sizes

Method of construction

$$\left[\left(\frac{d\vec{X}'}{dt}\right)_1 \quad \left(\frac{d\vec{X}'}{dt}\right)_2 \quad \cdots \quad \left(\frac{d\vec{X}'}{dt}\right)_n \right] = \mathbf{M}\left[\vec{X}_1' \quad \vec{X}_2' \quad \cdots \quad \vec{X}_n' \right]$$

Prescribed forcing (precisely known)

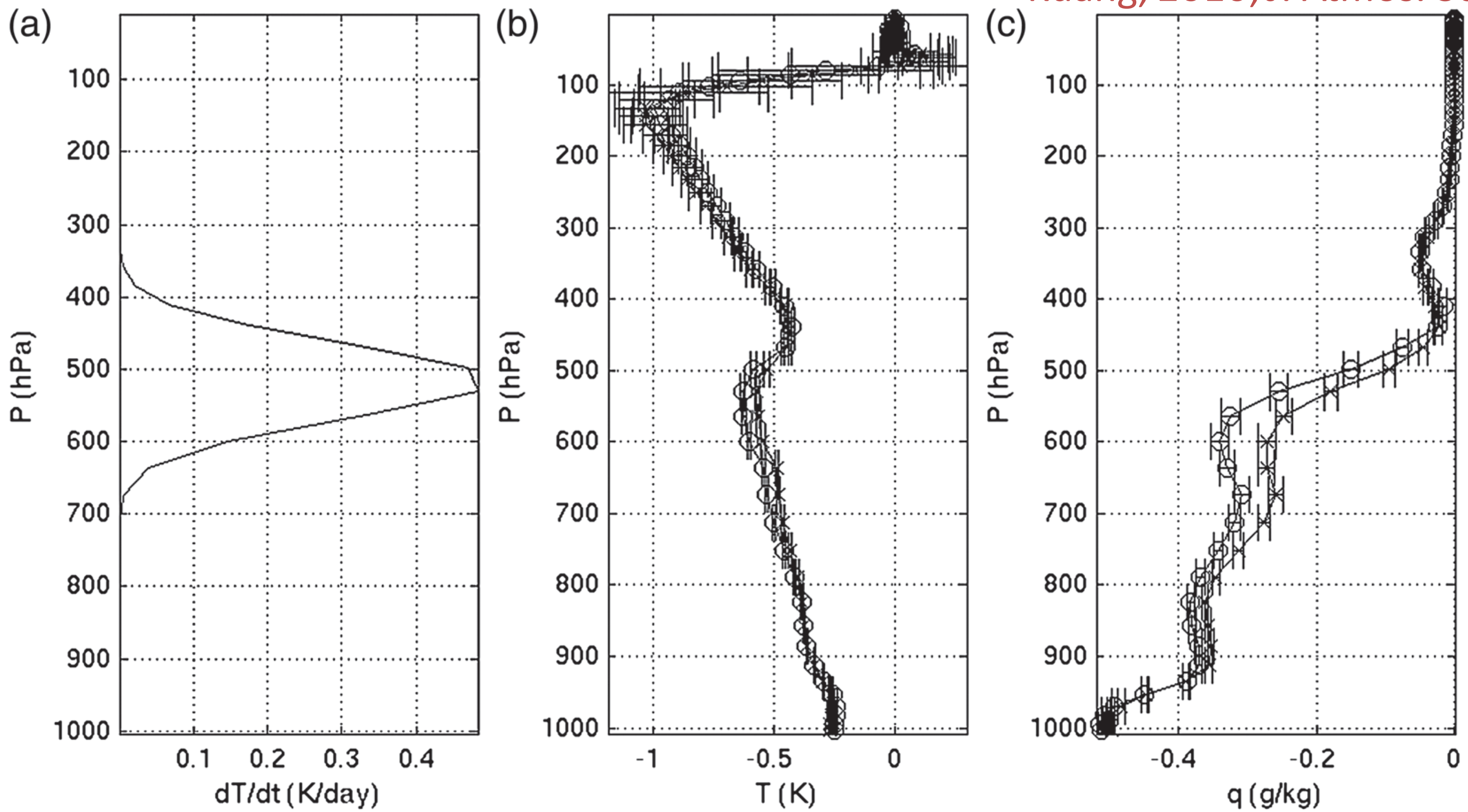
Equilibrium response X (has uncertainties)

Errors in eigenvalue λ: $|\delta\lambda| \propto |\lambda^2| \|\delta X\|$

- The fastest decaying modes of **M** (i.e. with the largest (in modulus) eigenvalues) have the largest errors
- The slowest decaying modes of **M** (i.e. the smallest eigenvalues) are the most accurate.
- The latter are of the most interest for coupling with large-scale flows

Linearity of convection!

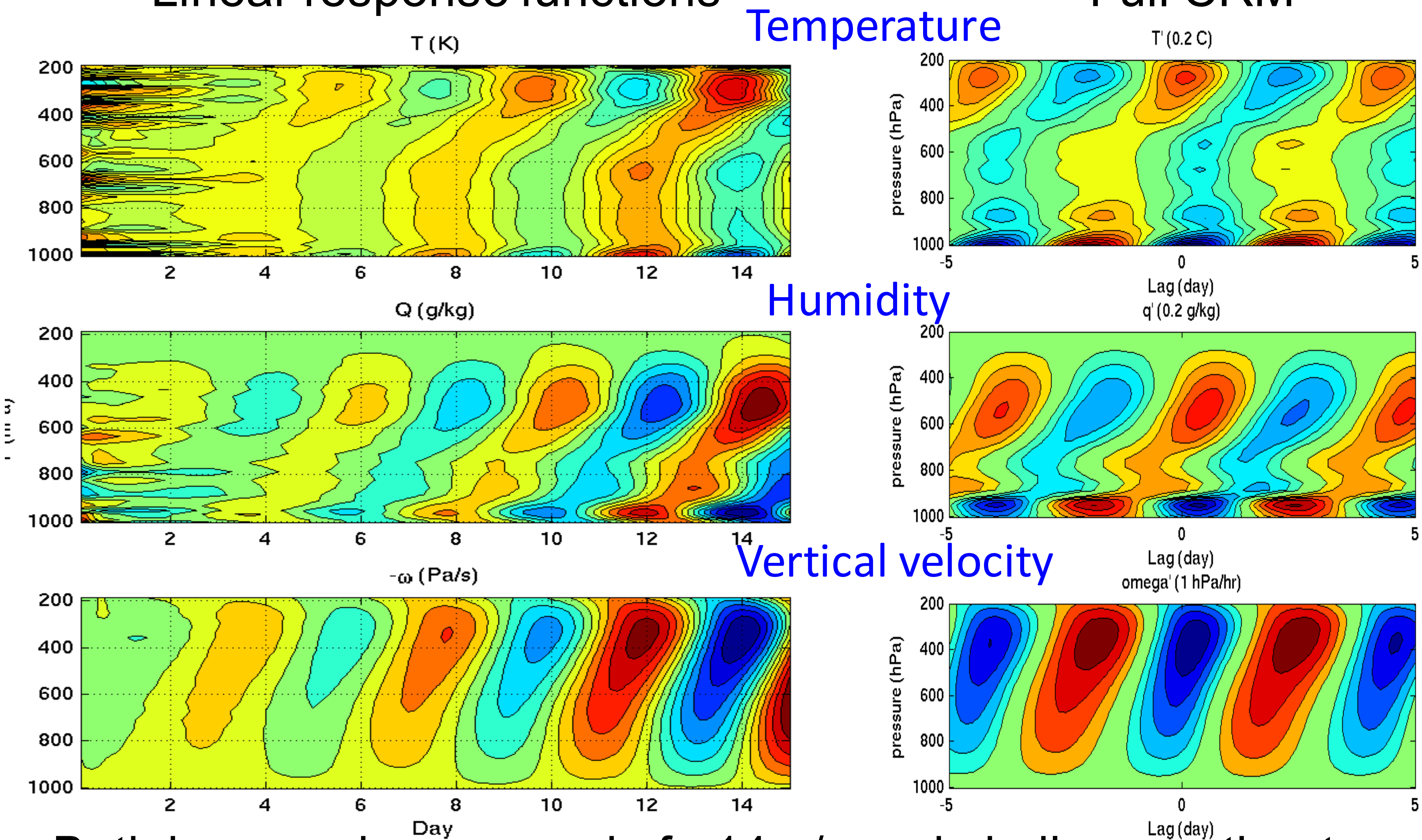
Kuang, 2010, J. Atmos. Sci.



Approximately linear. Combining the two to increase accuracy

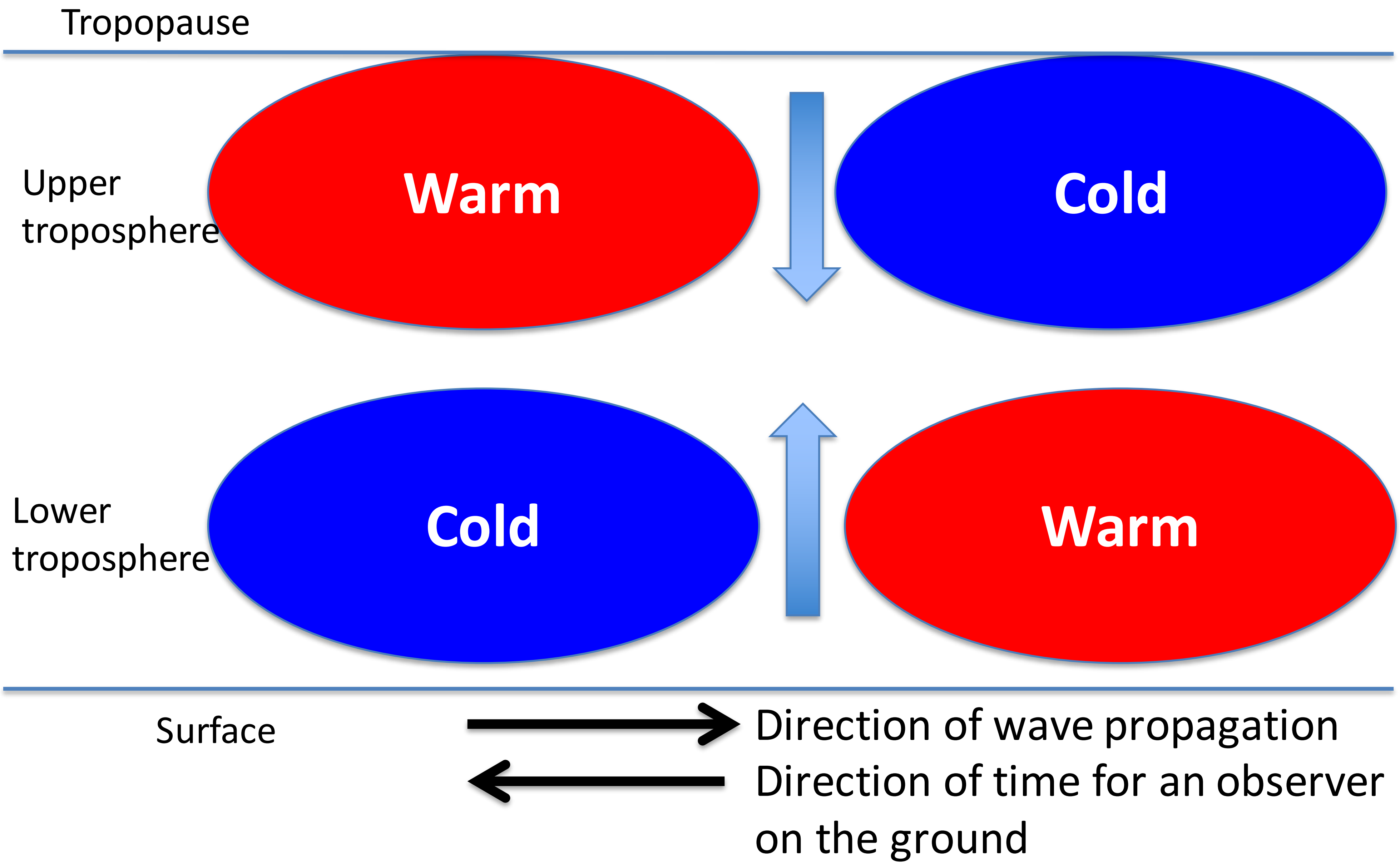
When coupled to 2D gravity wave

Linear response functions — Full CRM

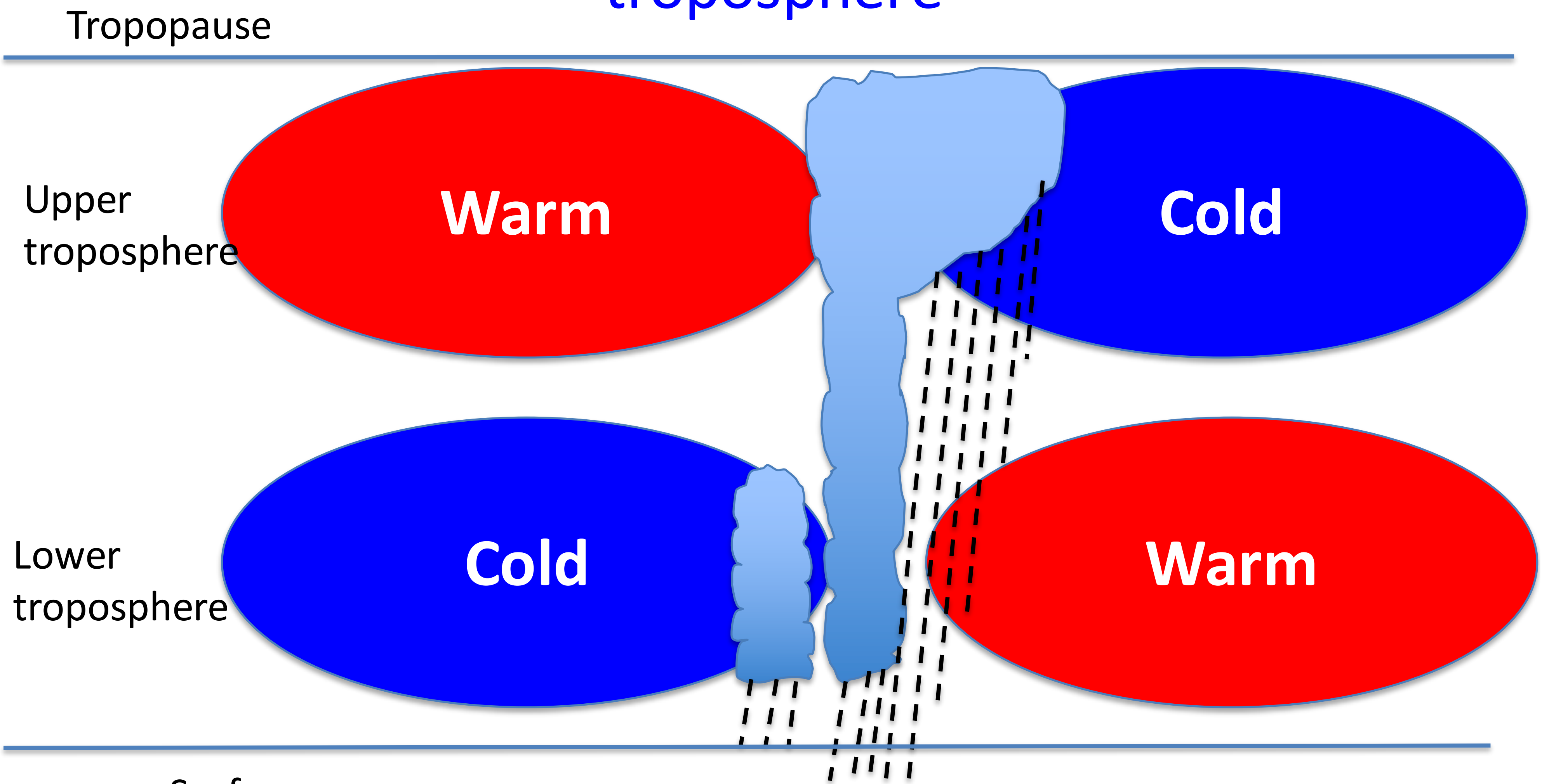


Both have a phase speed of ~14m/s and similar growth rates

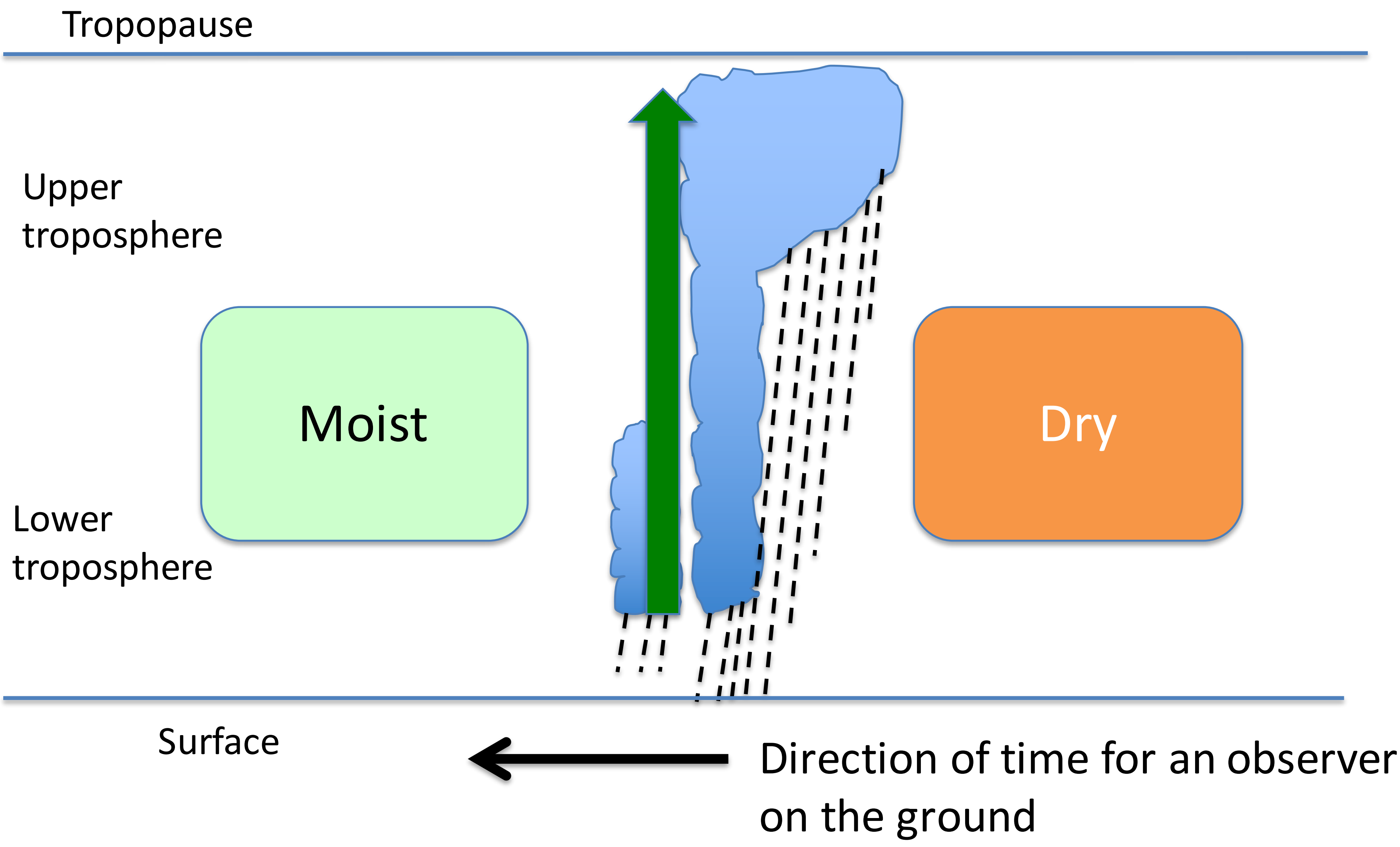
A gravity wave propagating to the right



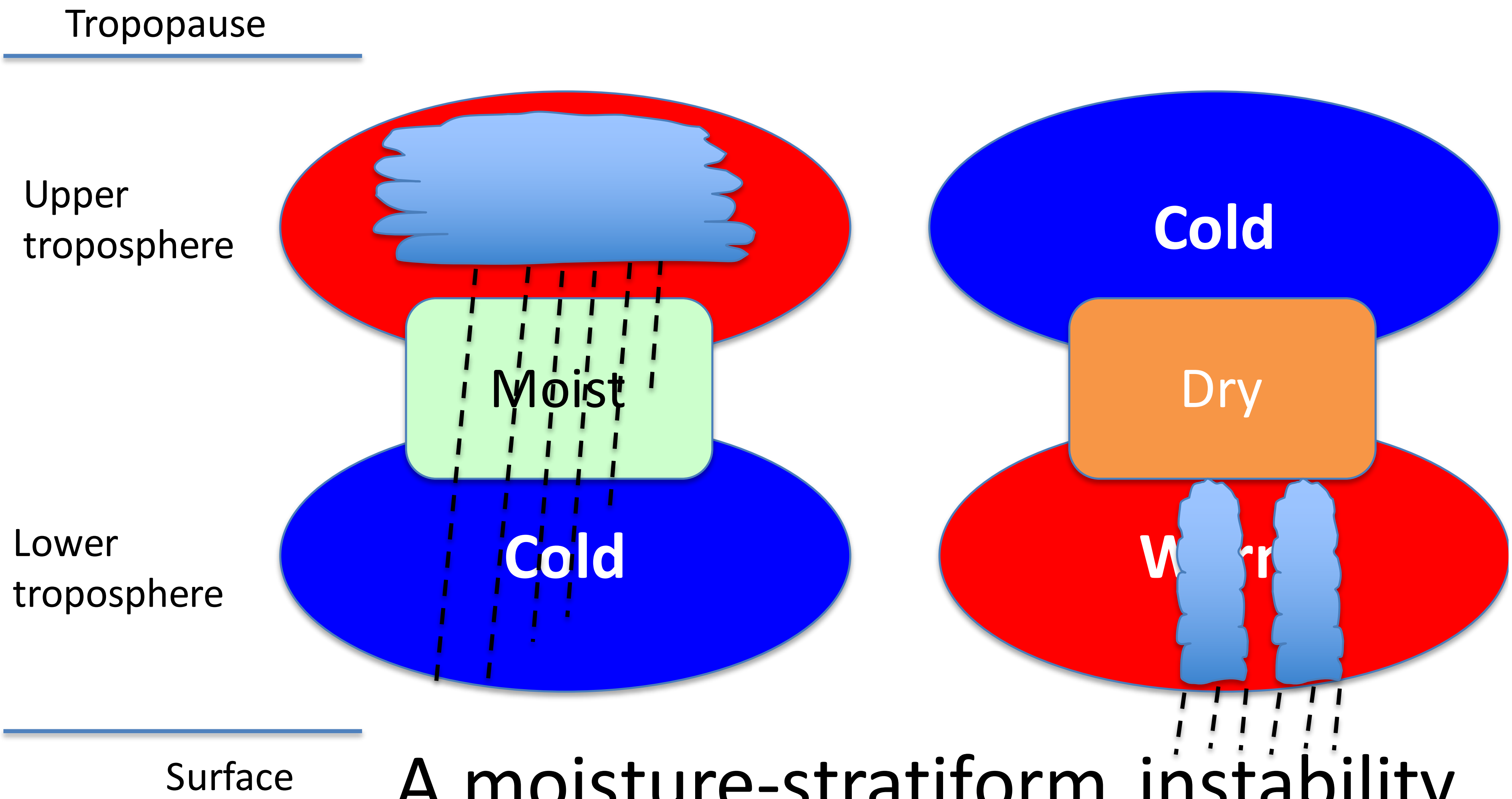
Convection responds to cooling of the lower troposphere



Deep convection and the associated vertical advection moisten the free troposphere



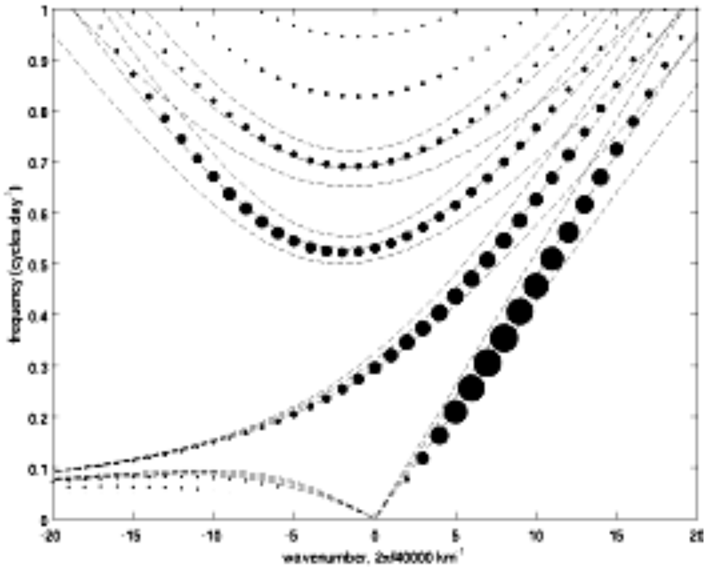
With a more moist free troposphere, convection reaches deeper



A moisture-stratiform instability

Extension to the equatorial beta plane

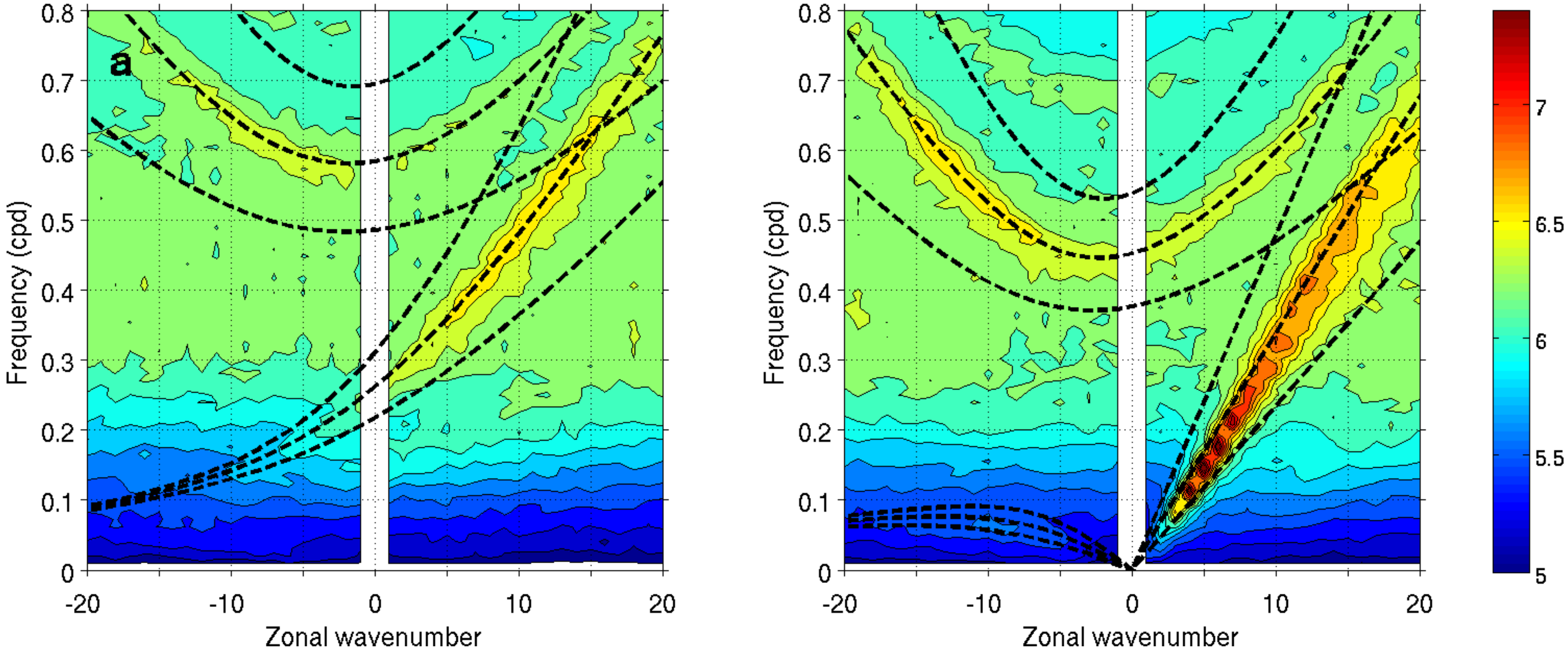
(Andersen and Kuang 2008)



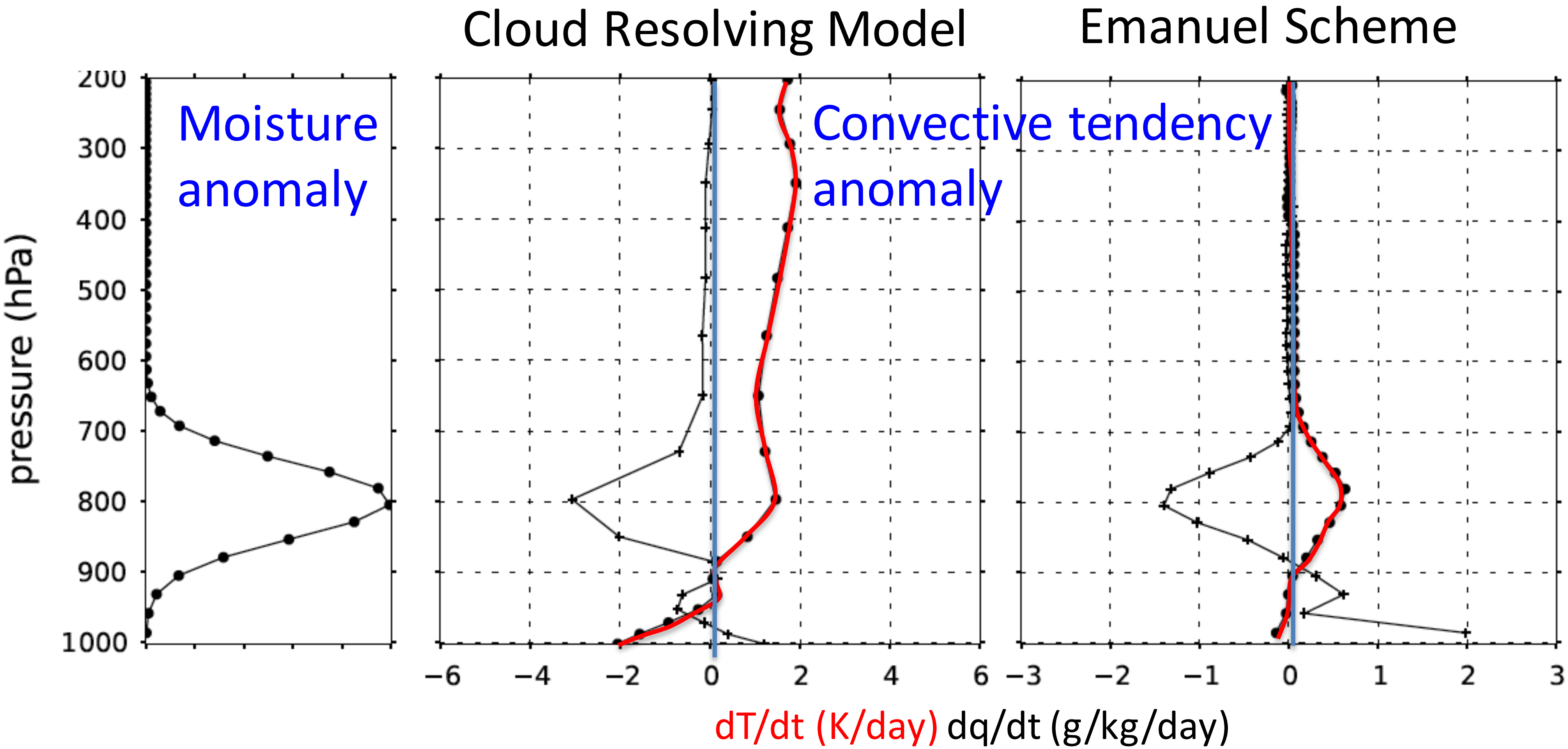
Dot size proportional to growth rate

Antisymmetric ($\log_{10}$(Power))

Symmetric ($\log_{10}$(Power))



Direct evaluations of the macroscopic behaviors of convective schemes

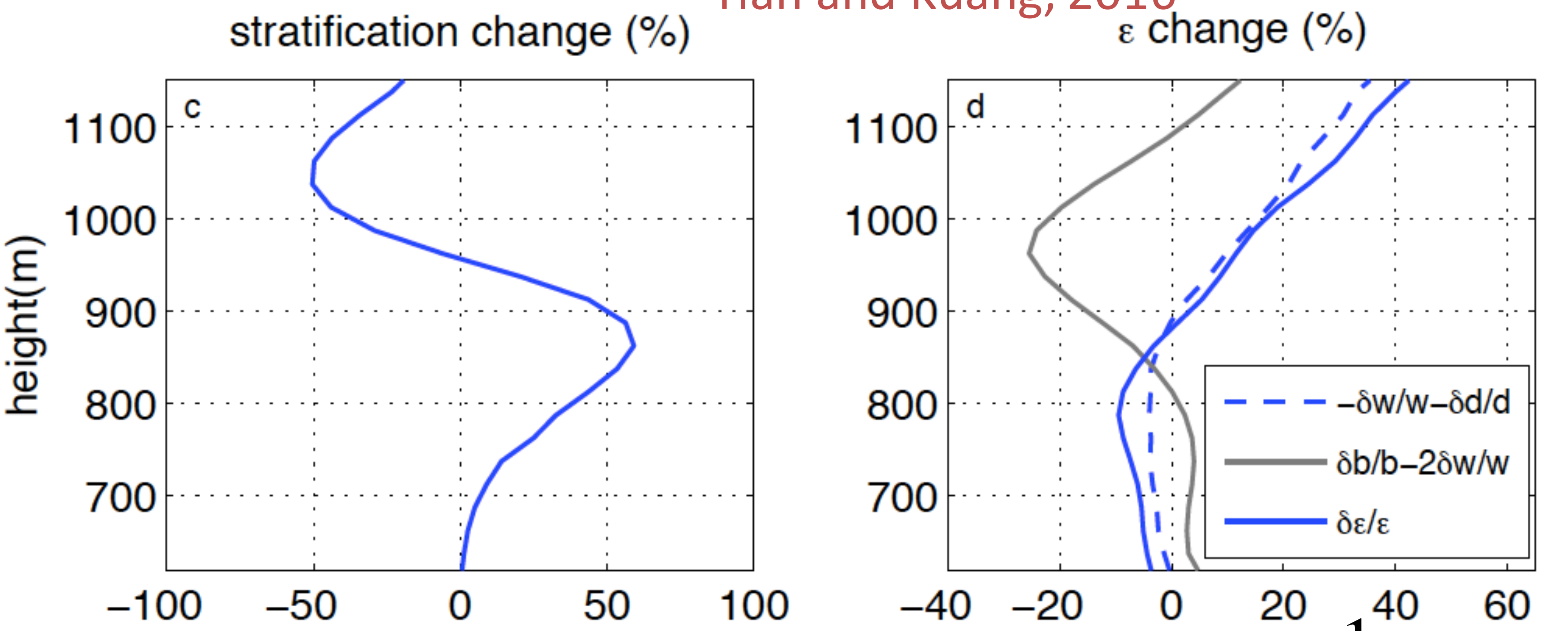


These comparisons offer clarity on why schemes don't produce convectively coupled tropical waves.

Herman and Kuang, 2013

Linear response functions can also help to constrain formulations of convective parameterizations

Tian and Kuang, 2016



Suggests fractional entrainment rate $\varepsilon \propto \frac{1}{wd}$

w is updraft vertical velocity, d is updraft size

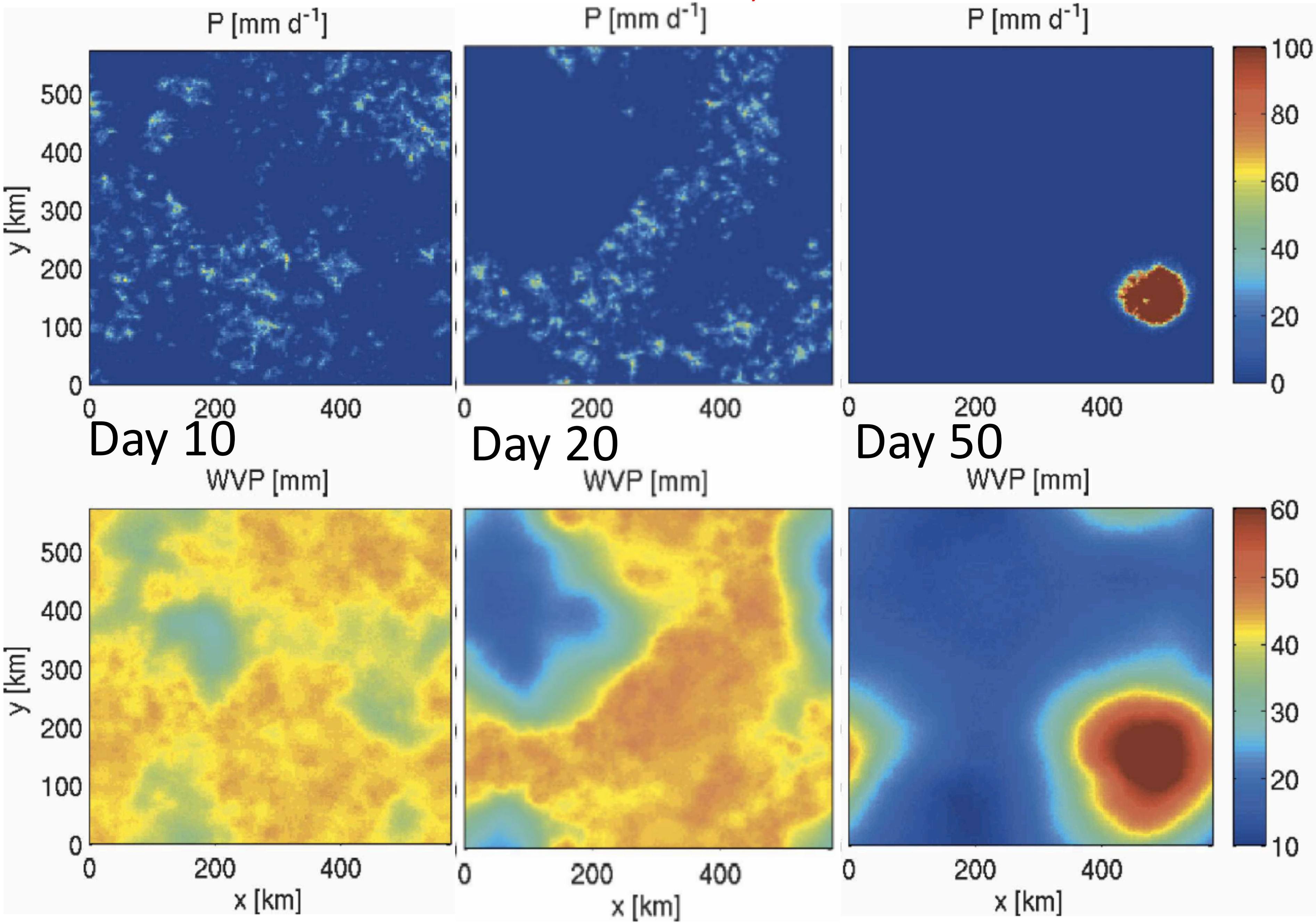
What about the Madden-Julian Oscillation?

Feedbacks from interactive surface heat flux and radiation appear essential.

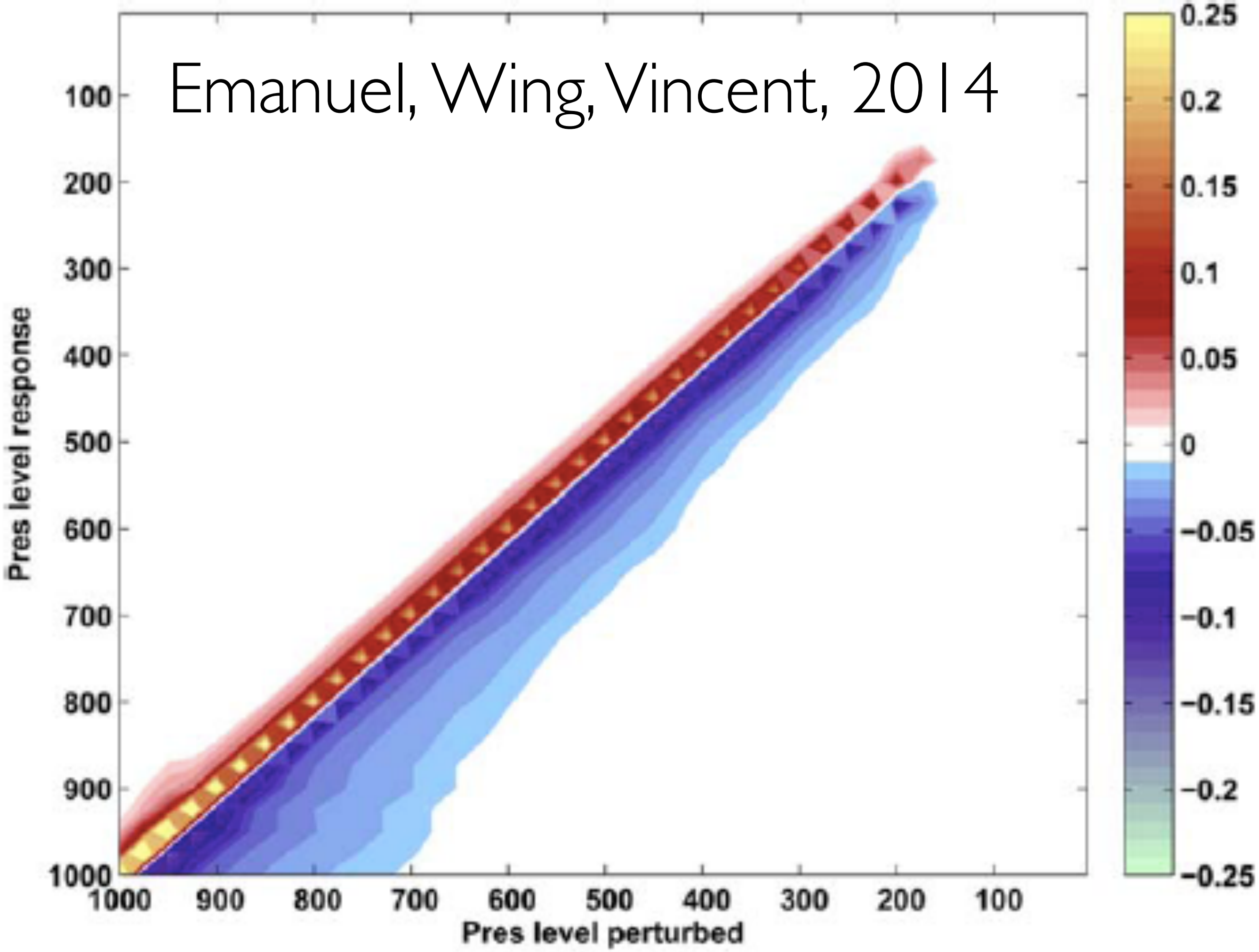
(Maloney, 2009; Kiranmayi and Maloney, 2011; Andersen and Kuang, 2012; Wu and Deng, 2013; Kim et al., 2014; Sobel et al., 2014; Arnold et al., 2015; Ma and Kuang 2016; among many others).

Convective self-aggregation

Bretherton et al., 2005



A linear radiative-convective instability for the initial phase of self-aggregation



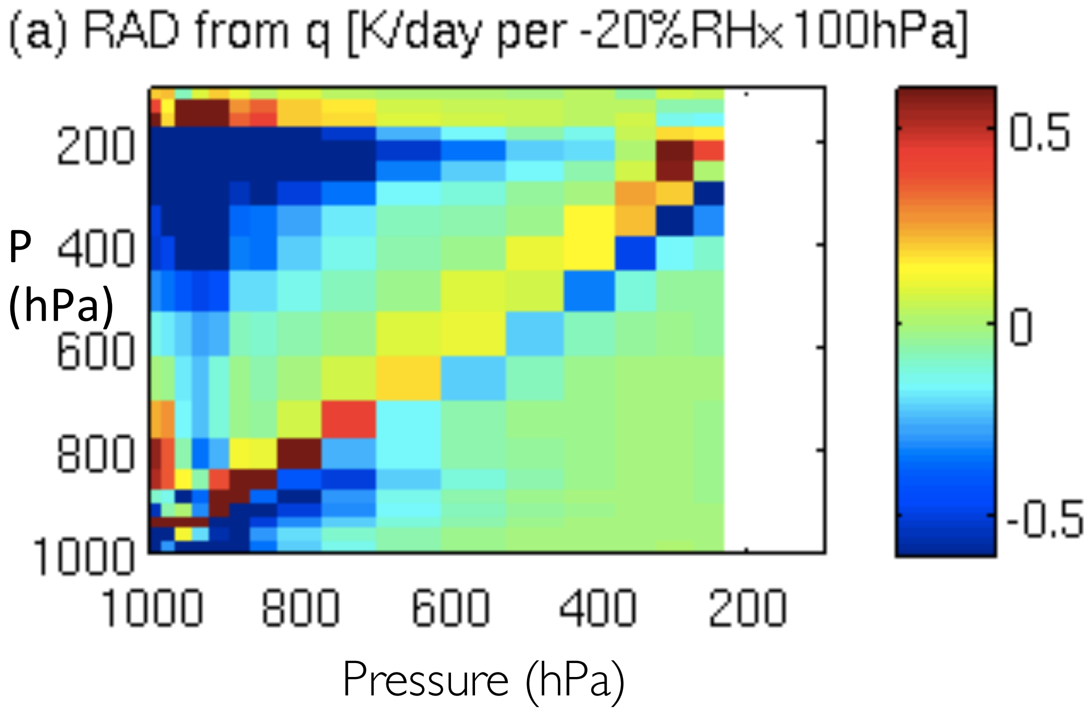
A linear radiative-convective instability for the initial phase of self-aggregation

[40] The instability of the two-layer model arises when downward motion dries both layers. The decreases emissivity of the upper layer leads to enhanced radiative cooling of the lower layer, which diminishes convection, leading to cooling of both layers and reinforcing the initial downward motion. As it is a linear model, the converse is also true, substituting upward for downward motion, and moistening for drying.

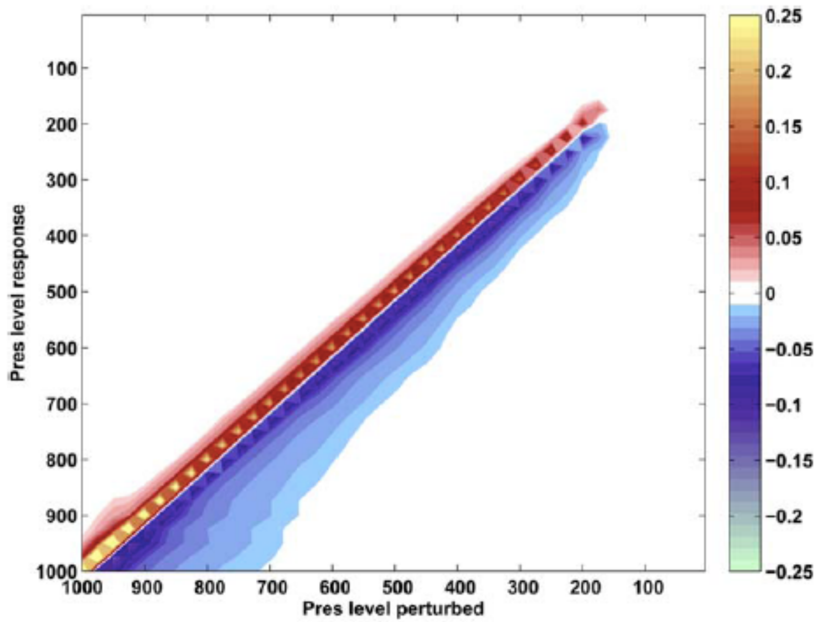
Emanuel, Wing, Vincent, 2014

Examine these ideas using linear response functions of a limited domain (128km by 128km) CRM

Radiative heating from a 20% reduction in RH

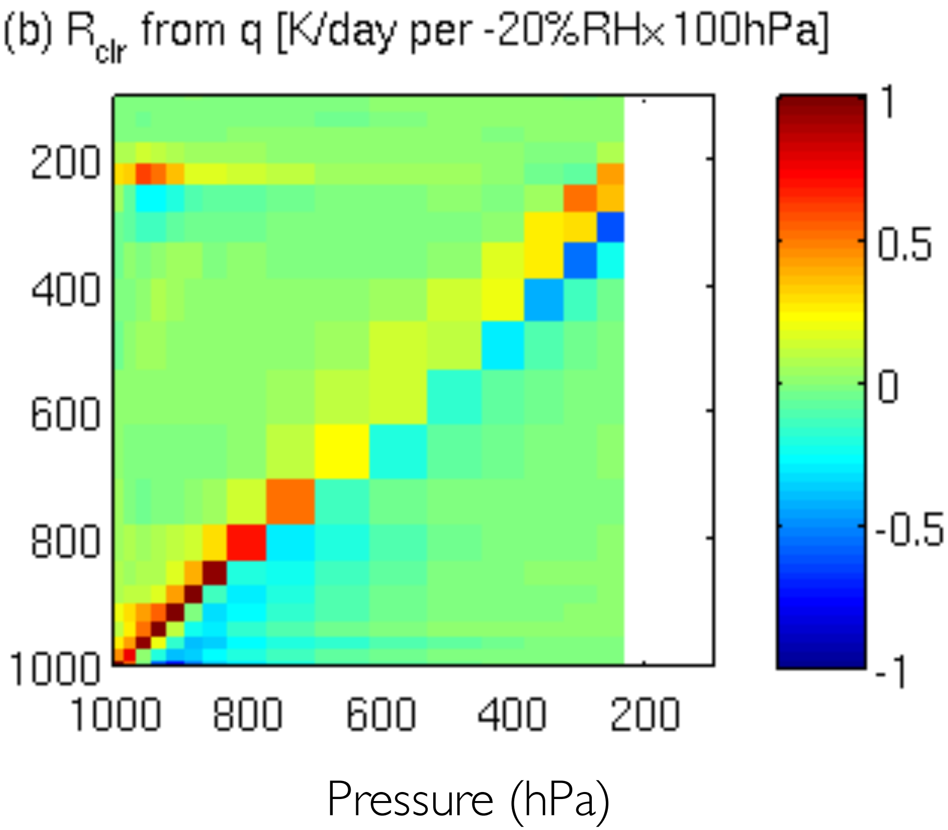


Left figure is for a fully developed cumulus field

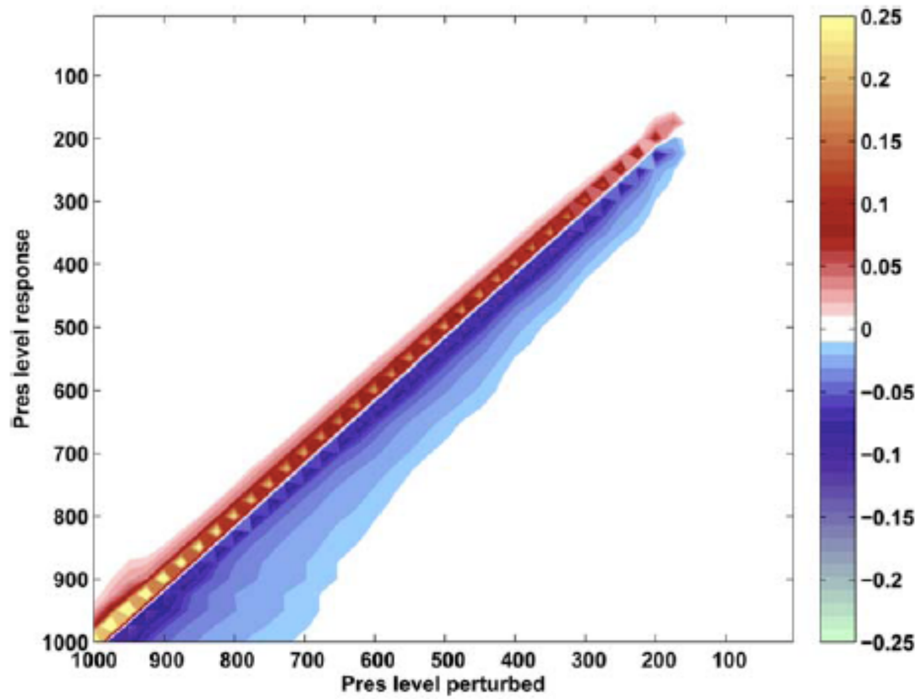


Right figure is from Emanuel et al., 2014 for RCE at 25C and 25hPa layers.

Radiative heating from a 20% reduction in RH

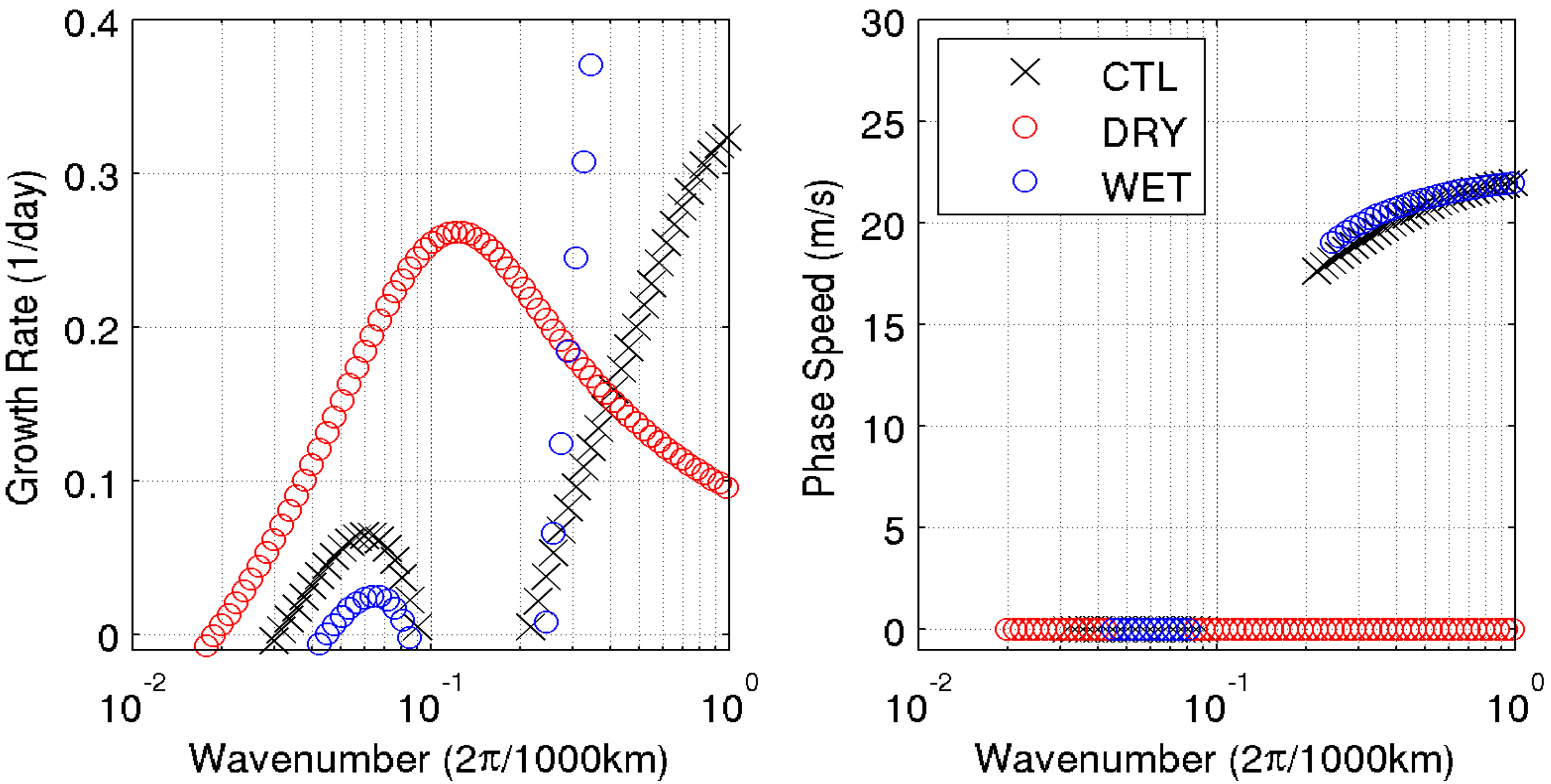


clear-sky radiation



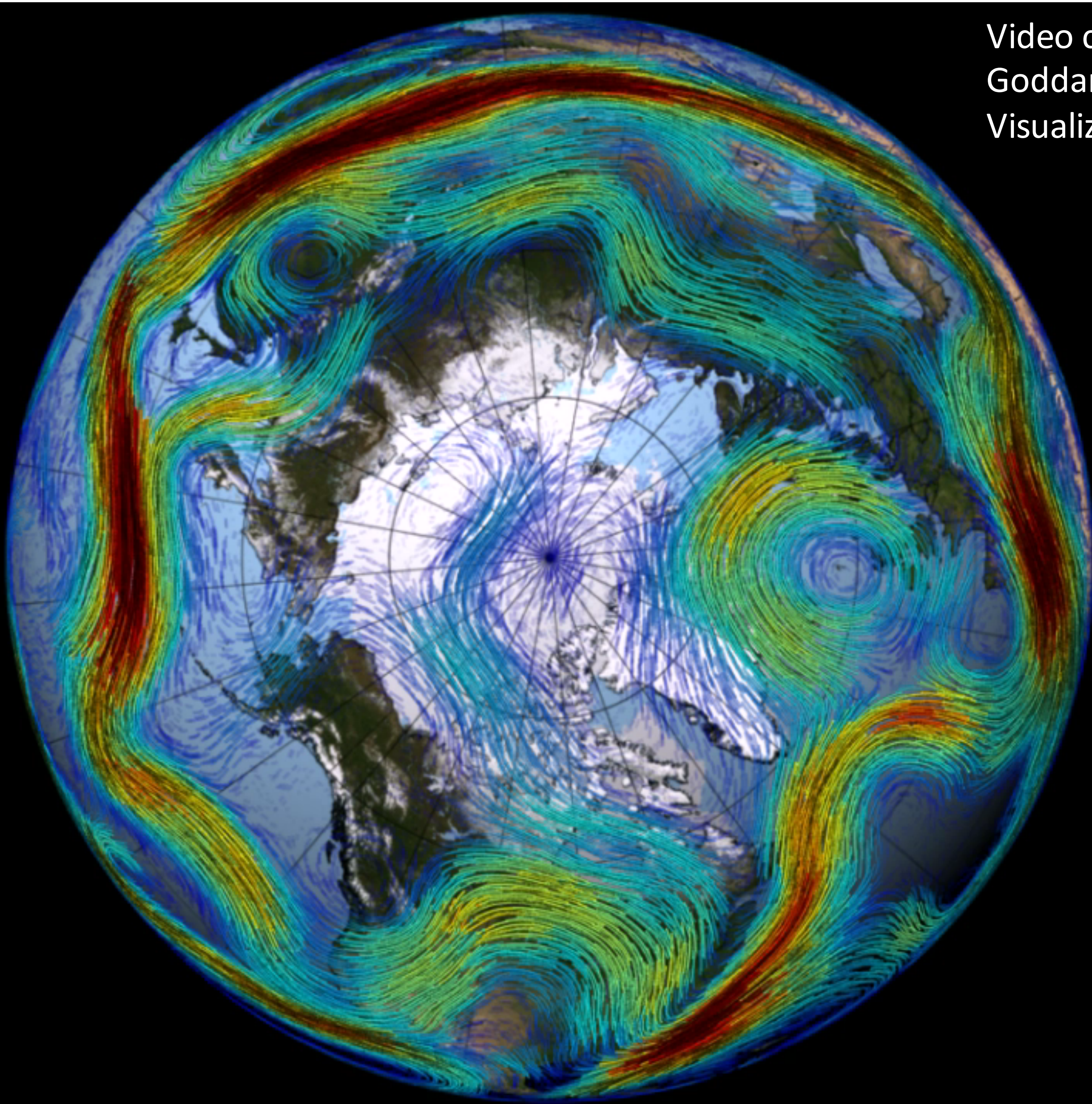
Right figure is from Emanuel et al., 2014, JAMES for RCE at 25C.

Couple linear response functions to linear gravity waves



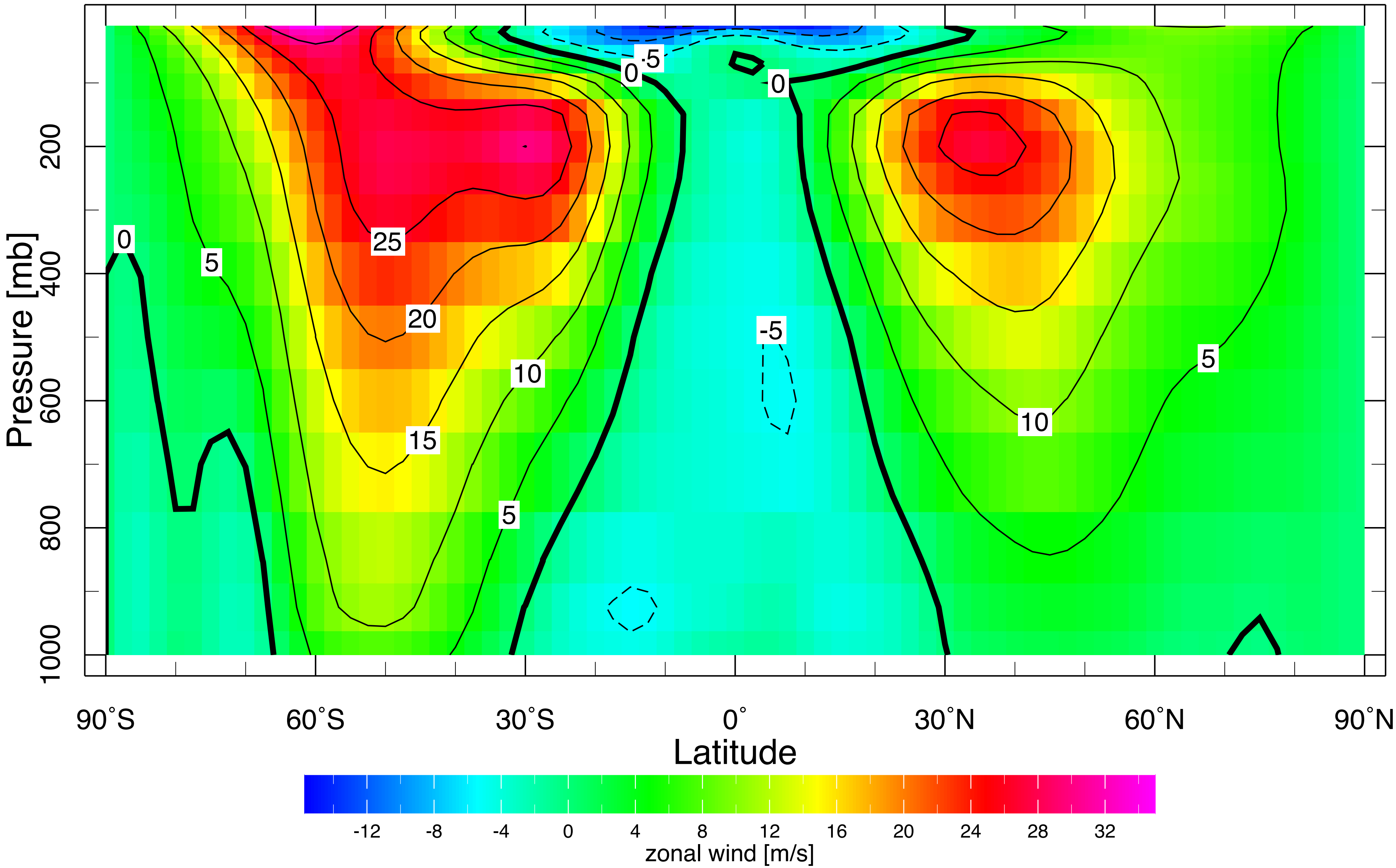
This could explain why dry patches dominate the growing phase of the self-aggregation. Kuang, in preparation

http://www.rantic.com/articles/social-media-tools-nail-hammer/

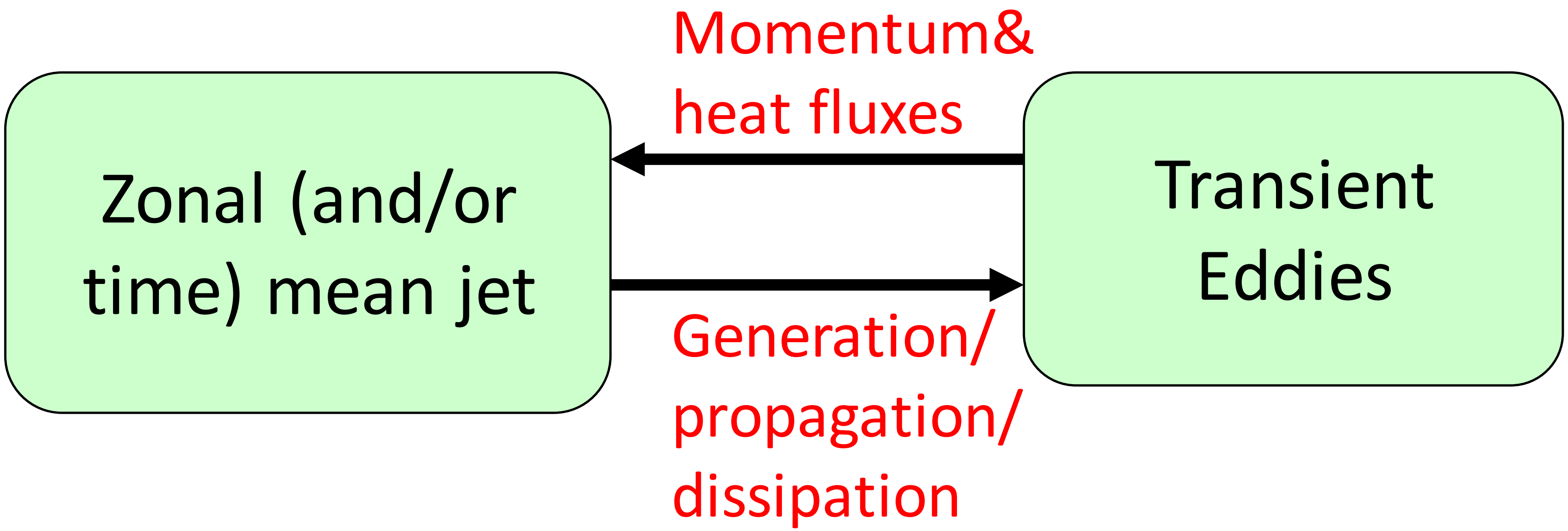


Video credit: NASA Goddard Scientific Visualization Studio

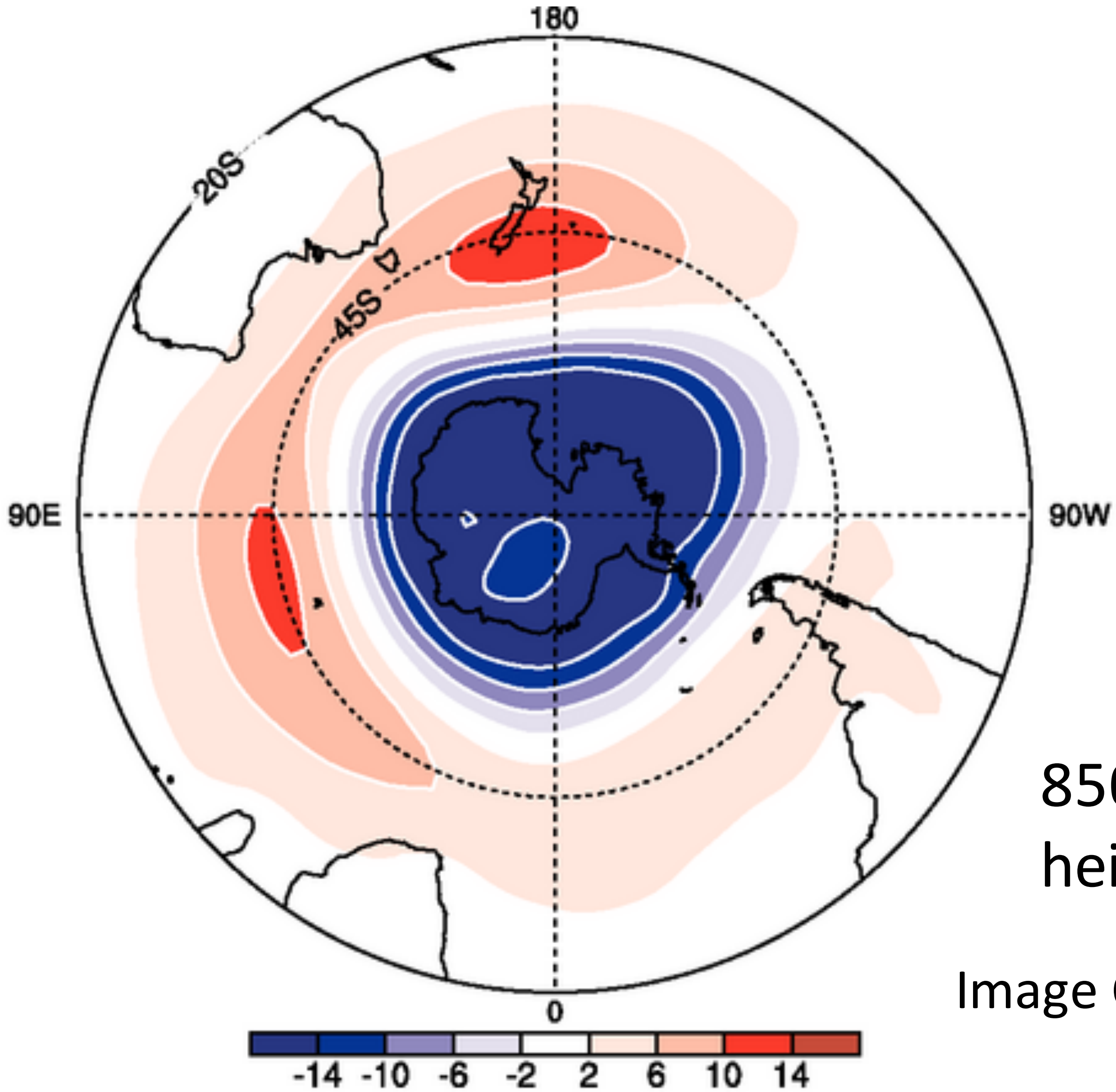
Climatology of zonal wind



Jet dynamics



The strong coupling between eddies and the jet is key to the jet dynamics.

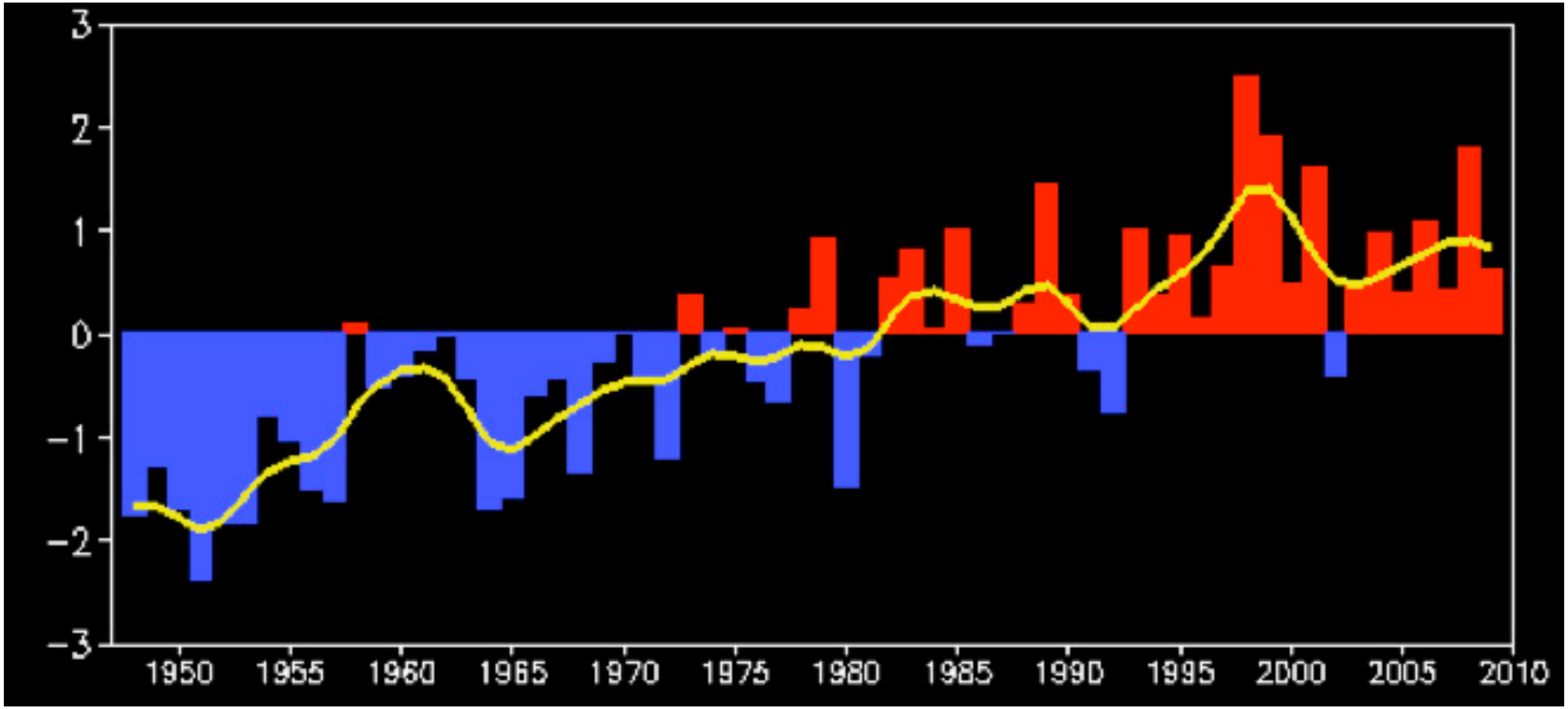


Southern annular mode

850hPa Geopotential height

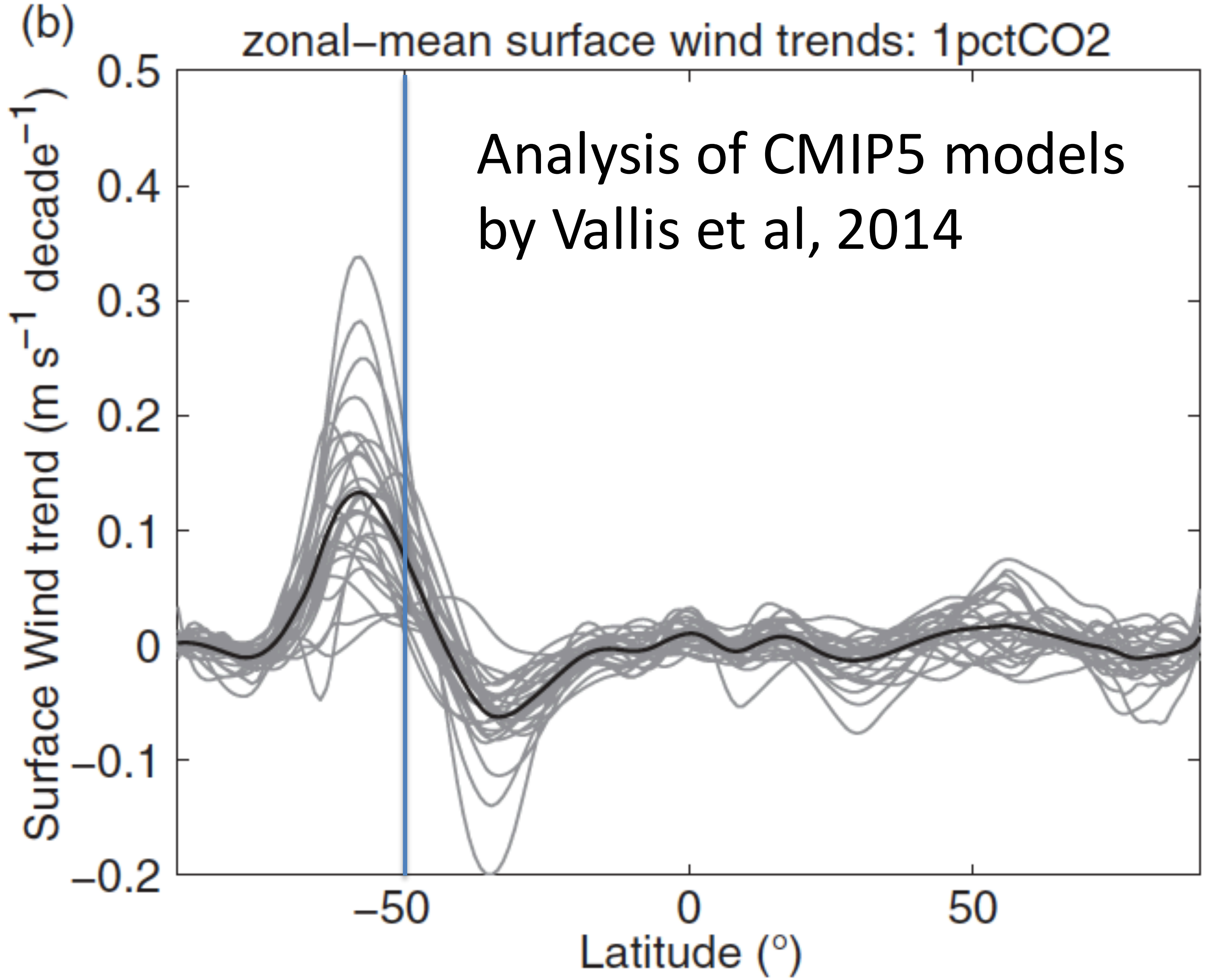
Image Credit: IPCC AR4 report

Trend in Southern Annular Mode index

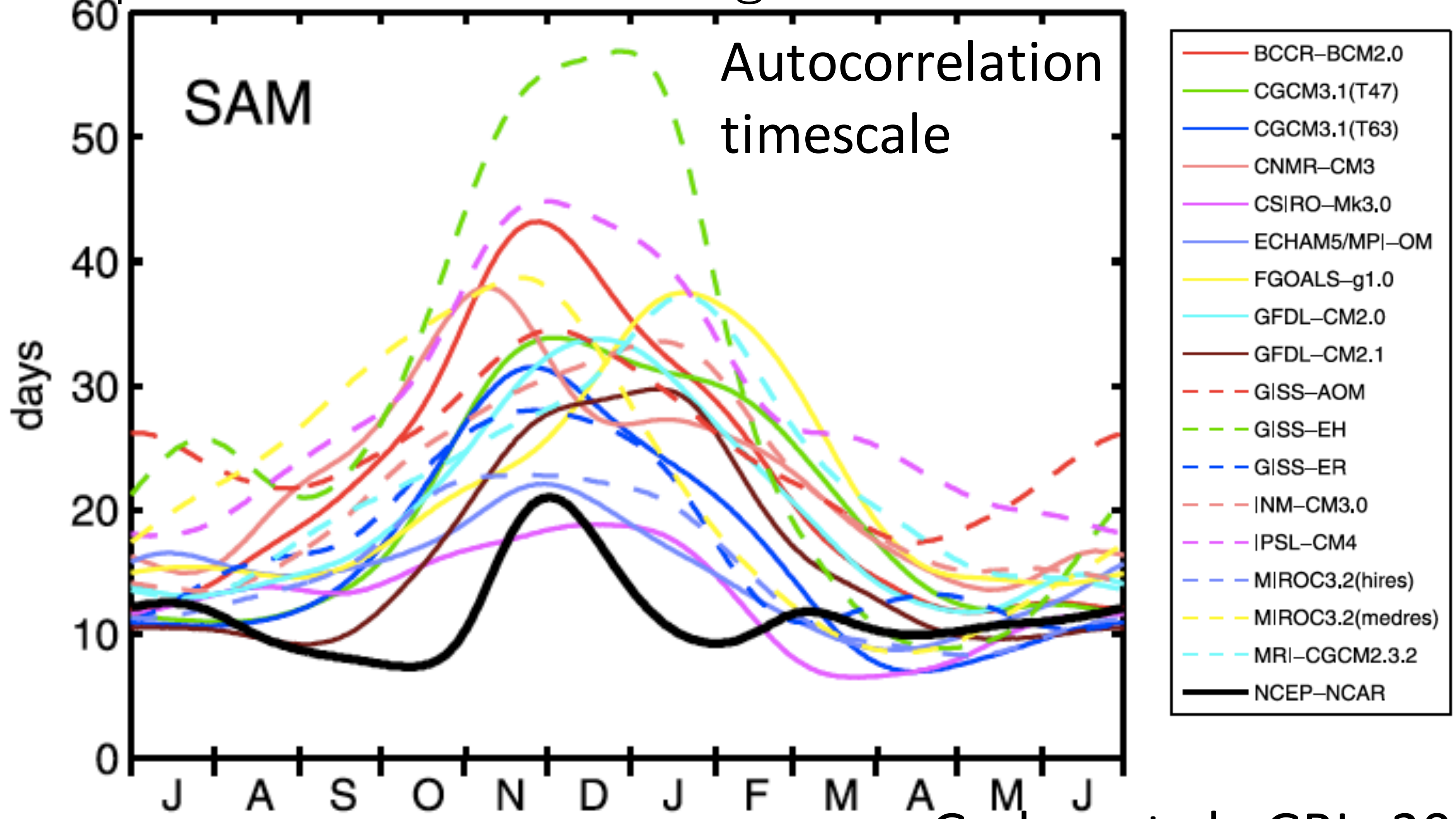


Credit: Jianping Li
See also, Kwok and Comiso, 2002

Projected pole-ward shift of the jet



Annular modes are too persistent in IPCC models, suggesting the models may also overestimate the response to climate forcing



Gerber et al., GRL, 2008

How to quantify the eddy-jet feedback?

A simple model by Lorenz and Hartmann (2001)

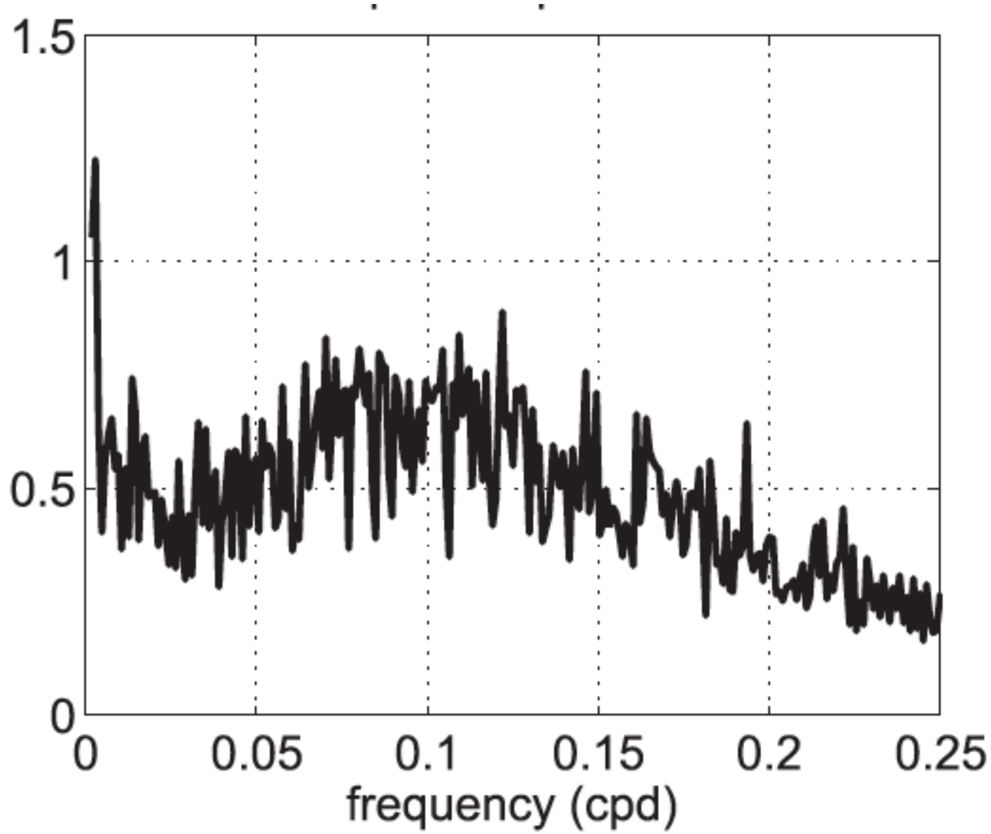
$$\frac{dz}{dt} = -\frac{z}{\tau} + F_{eddy}$$

$$F_{eddy} = bz + \xi$$

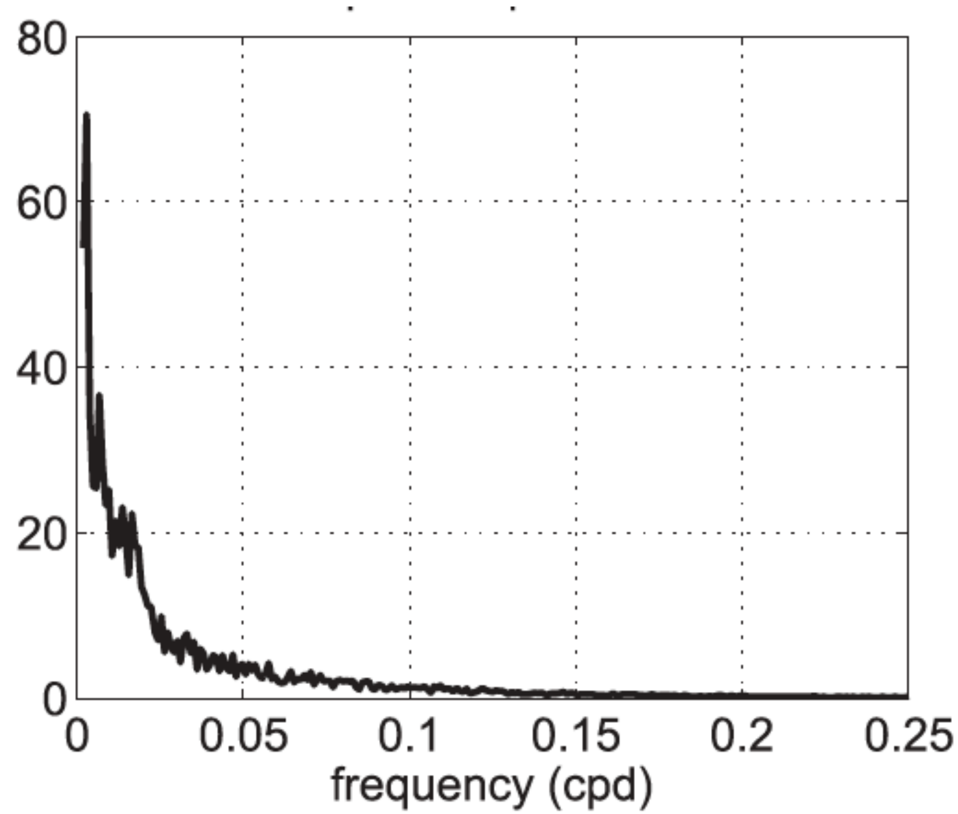
← Random noise independent of z

Z: the zonal index F_{eddy}: eddy forcing

Power spectrum of eddy forcing F_{eddy}

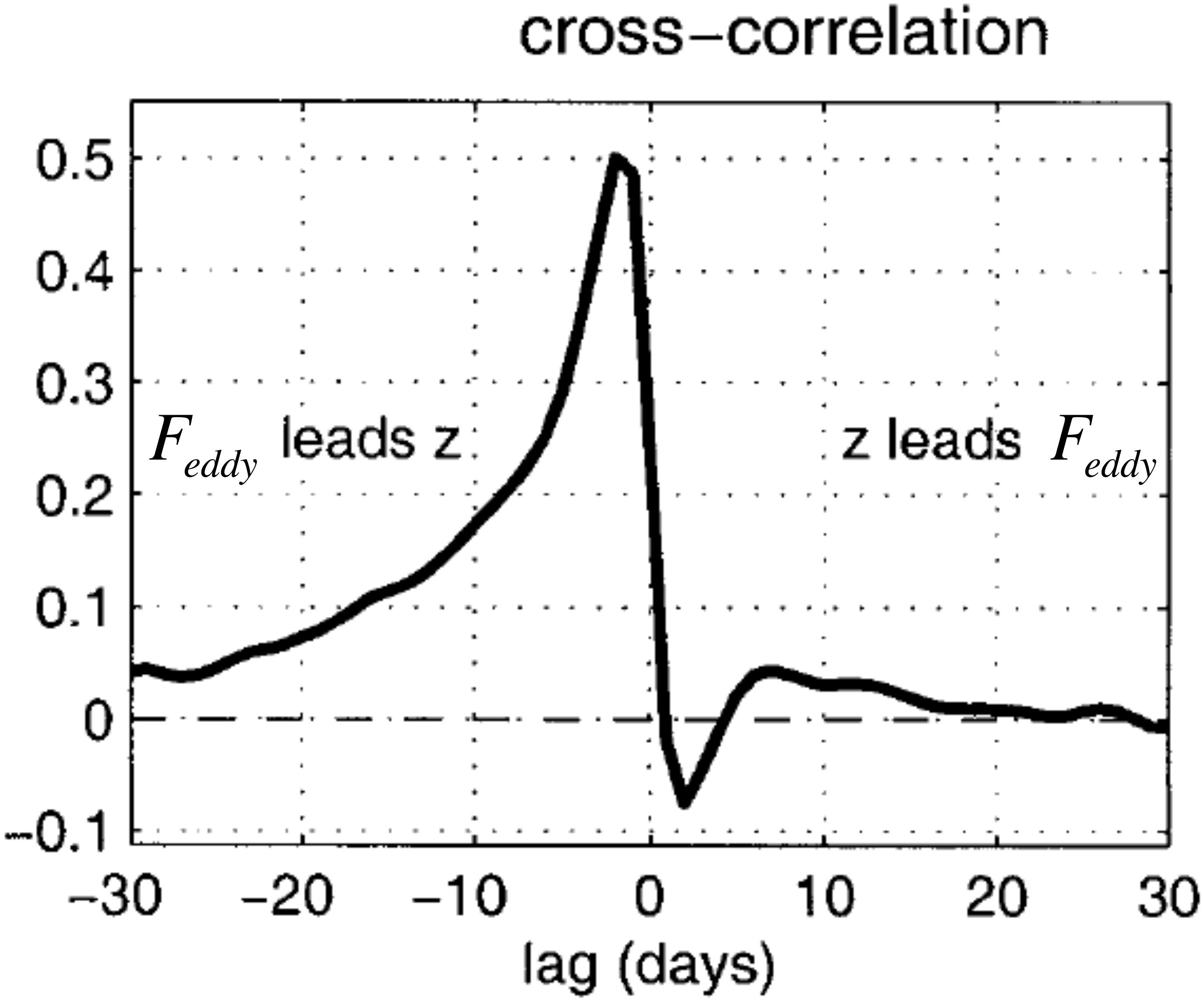


Power spectrum of zonal index Z



Lorenz and Hartmann (2001)

Lorenz and Hartmann (2001)



Contemporaneous regression doesn't give an estimate of the feedback

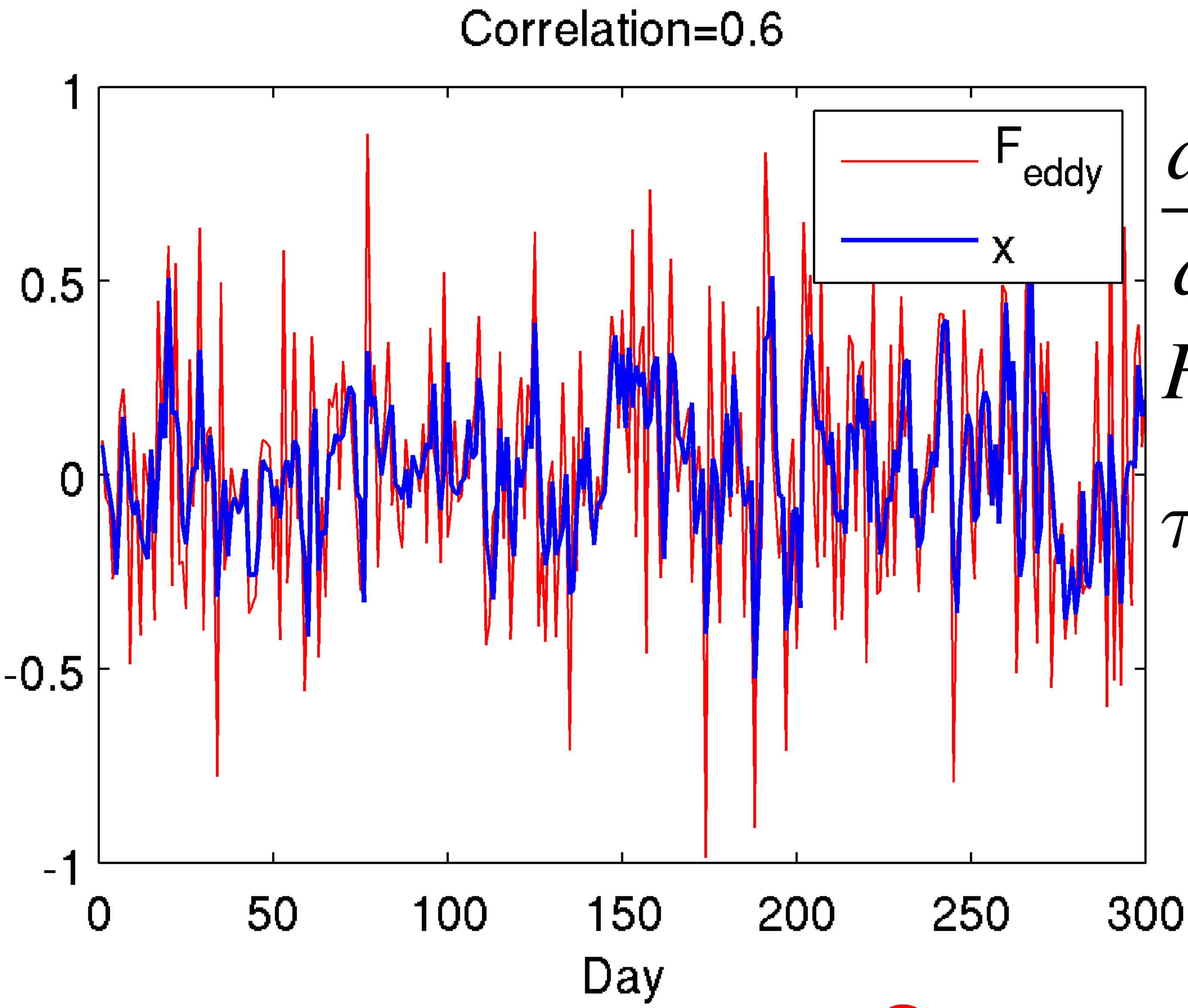
Let's look at a simple example

$$\frac{dx}{dt} = -\frac{x}{\tau} + F_{eddy}$$

$$F_{eddy} = ax + \xi$$

Random noise

$$\tau = 1 \ day$$



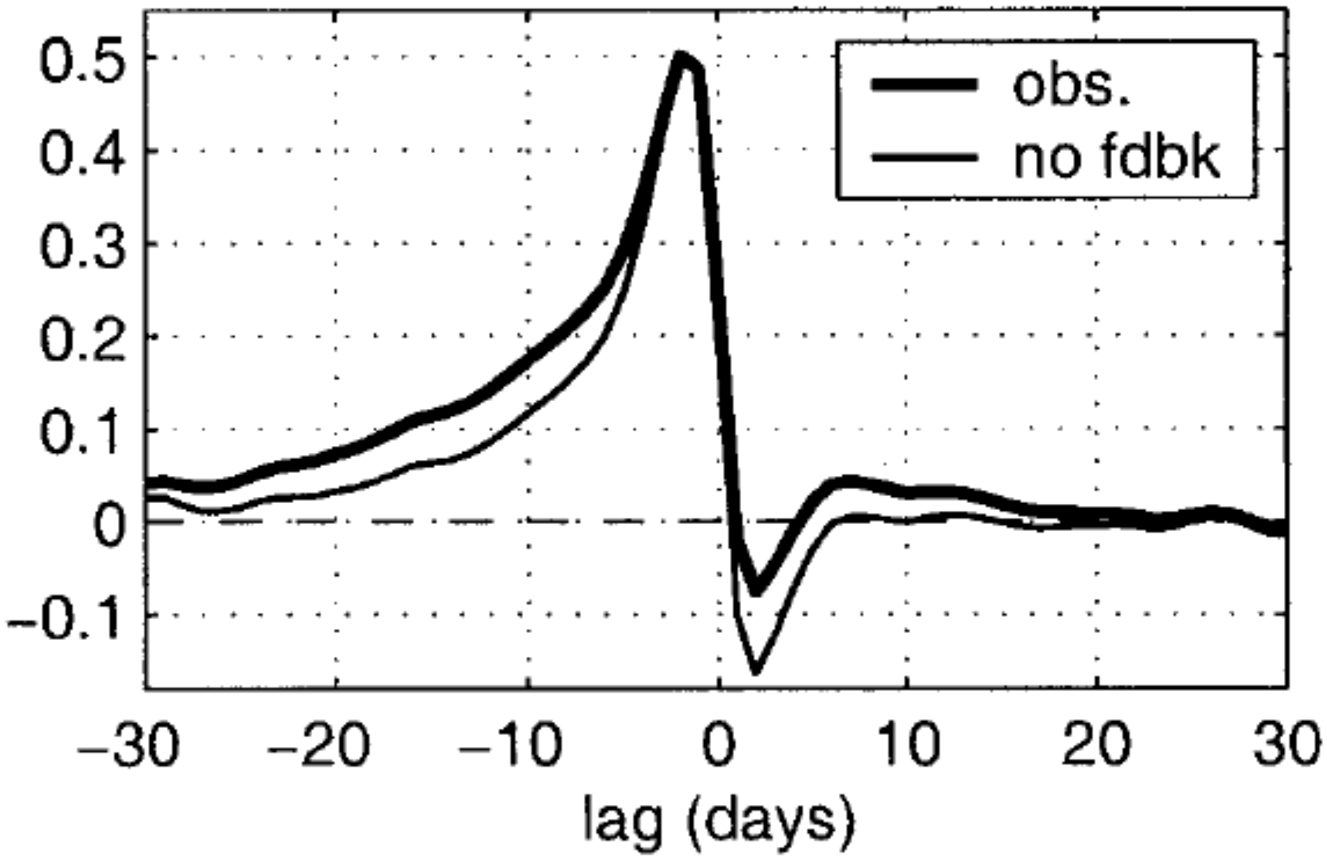
$$\frac{dx}{dt} = -\frac{x}{\tau} + F_{eddy}$$

$$F_{eddy} = ax + \xi$$

$$\tau = 1 \ \ day$$

Generated with a=0

Lorenz and Hartmann (2001)



Key assumption: random (or mean-state independent) eddy forcing and zonal index decorrelate at long (positive) lags

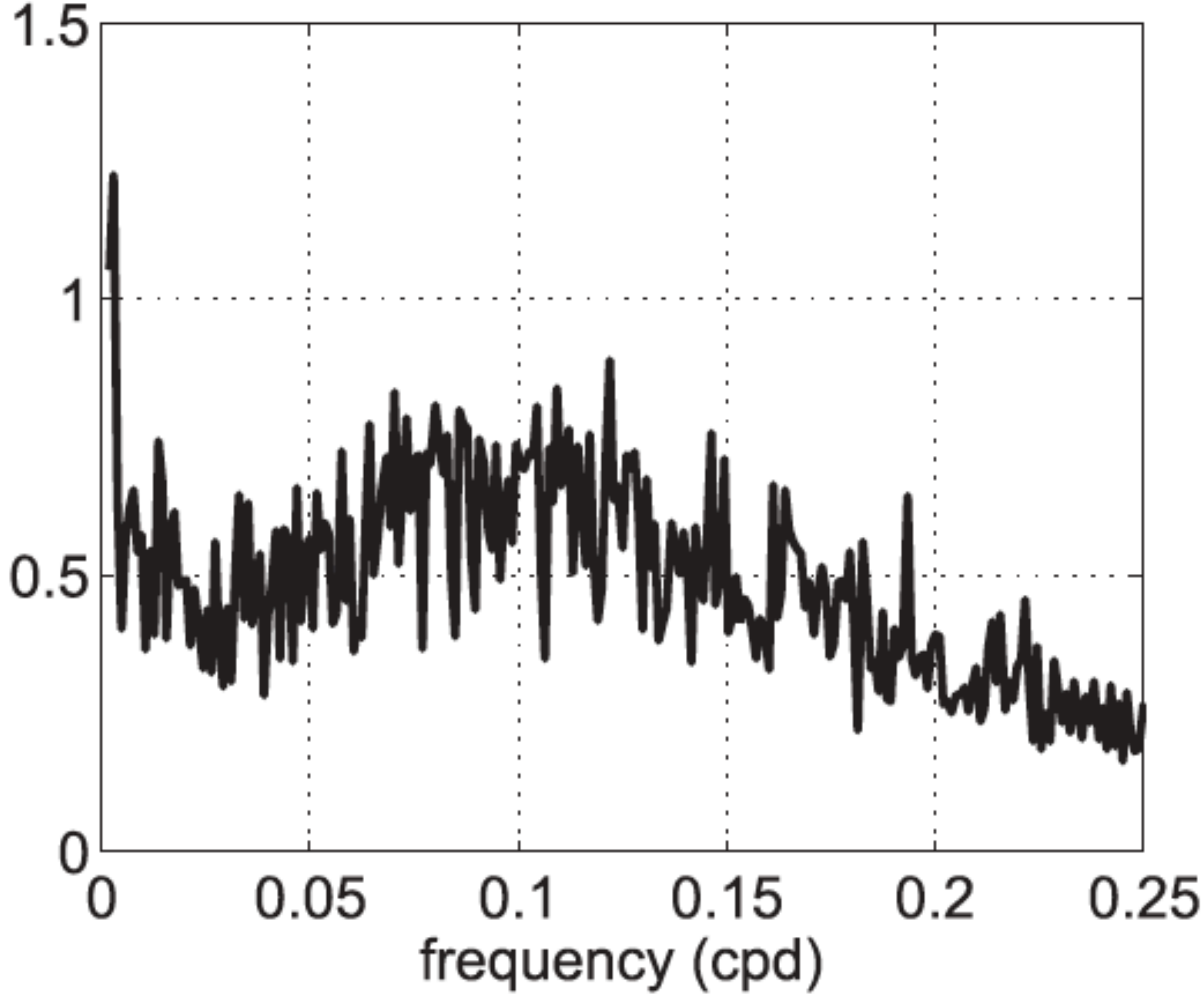
Simpson et al. (2013)

$$b = \frac{corr_l\left(z, F_z\right)}{corr_l\left(z, z\right)}$$

$corr_l(x,y)$ is the lag correlation when x leads y by l.

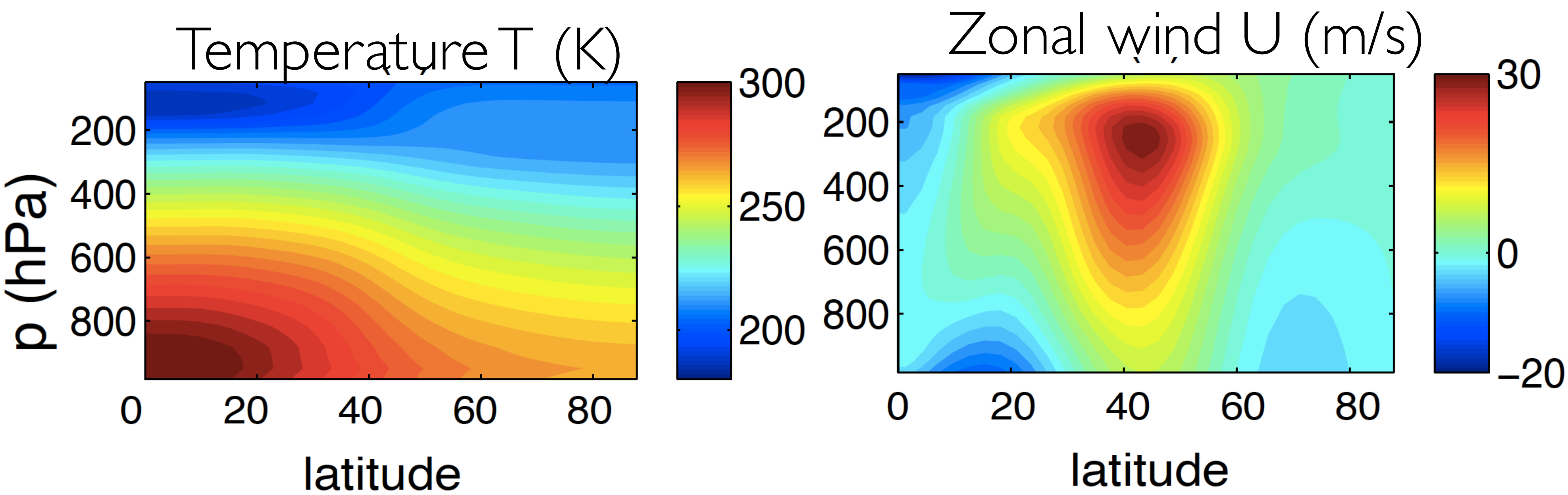
Key assumption: random (or mean-state independent) eddy forcing and zonal index are uncorrelated at lag l.

Power spectrum of the eddy forcing has a broad peak, and the previous assumptions may not hold.



Test these approaches in a simple dry general circulation model where the linear response functions can provide the ground truth.

Atmosphere-only (zonally symmetric forcing, no ocean, ice, snow, topography, or seasonal cycle …)



Linear response functions

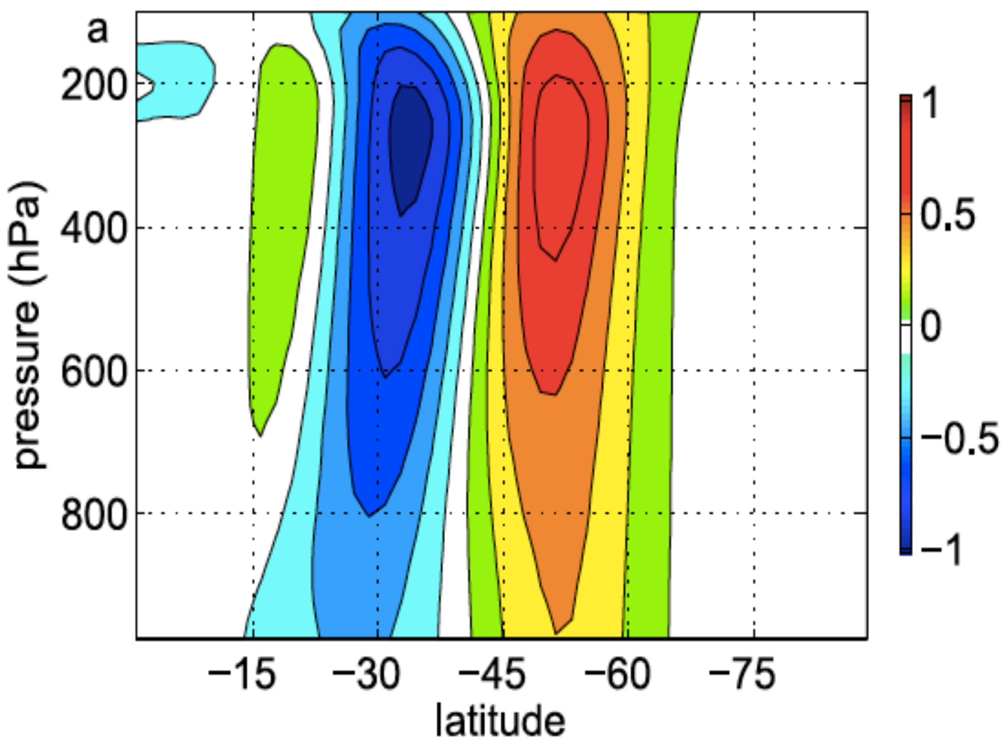
$$\frac{d\vec{X}'}{dt} = \mathbf{M}\vec{X}' + \vec{F}'$$

- Define the (mean field) state vector $\vec{X}'$ to include include anomalous zonal mean zonal wind U and zonal mean temperature T
- This equation assumes that
  - Zonal mean T, U completely describe the state of the atmosphere, i.e. baroclinic eddies are in statistical equilibrium with the T, U distributions. Reasonable for phenomena with timescales of days or more.
  - Linearity holds for perturbations of relevant sizes

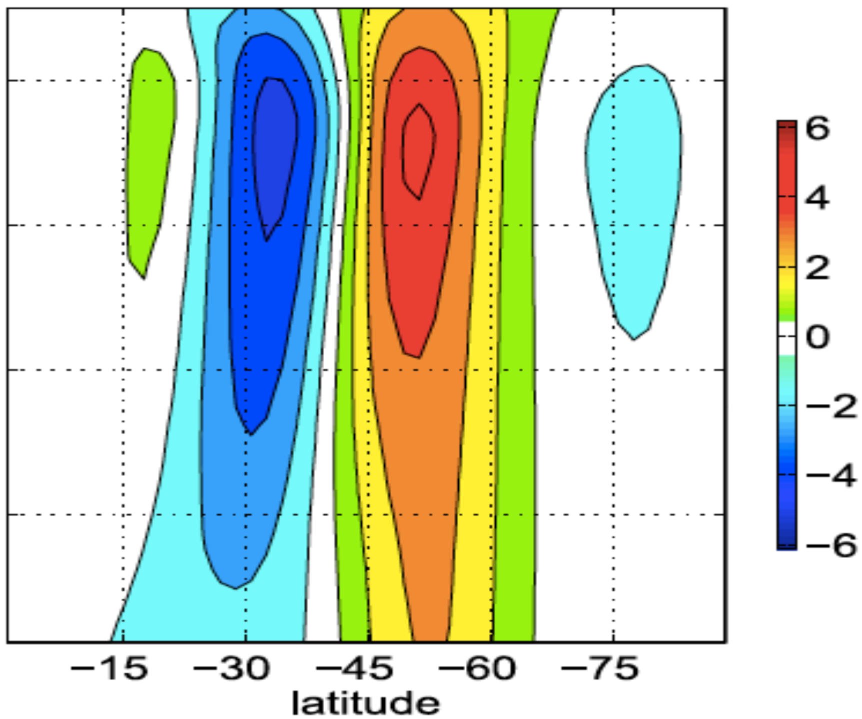
See Hassanzadeh and Kuang (2016ab) for details

"Perpetual annular mode"

Forced annular mode pattern

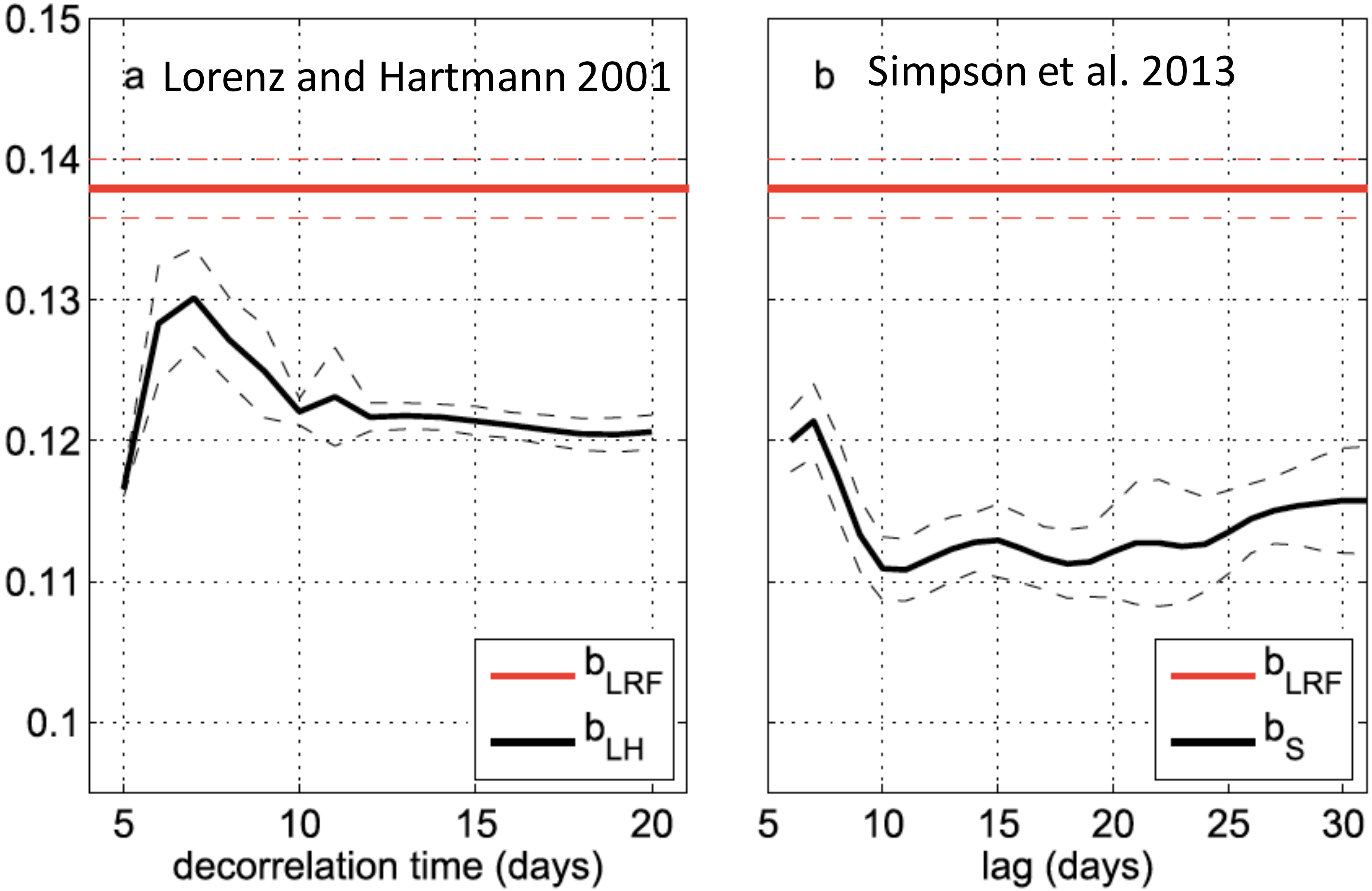


Internal annular mode pattern



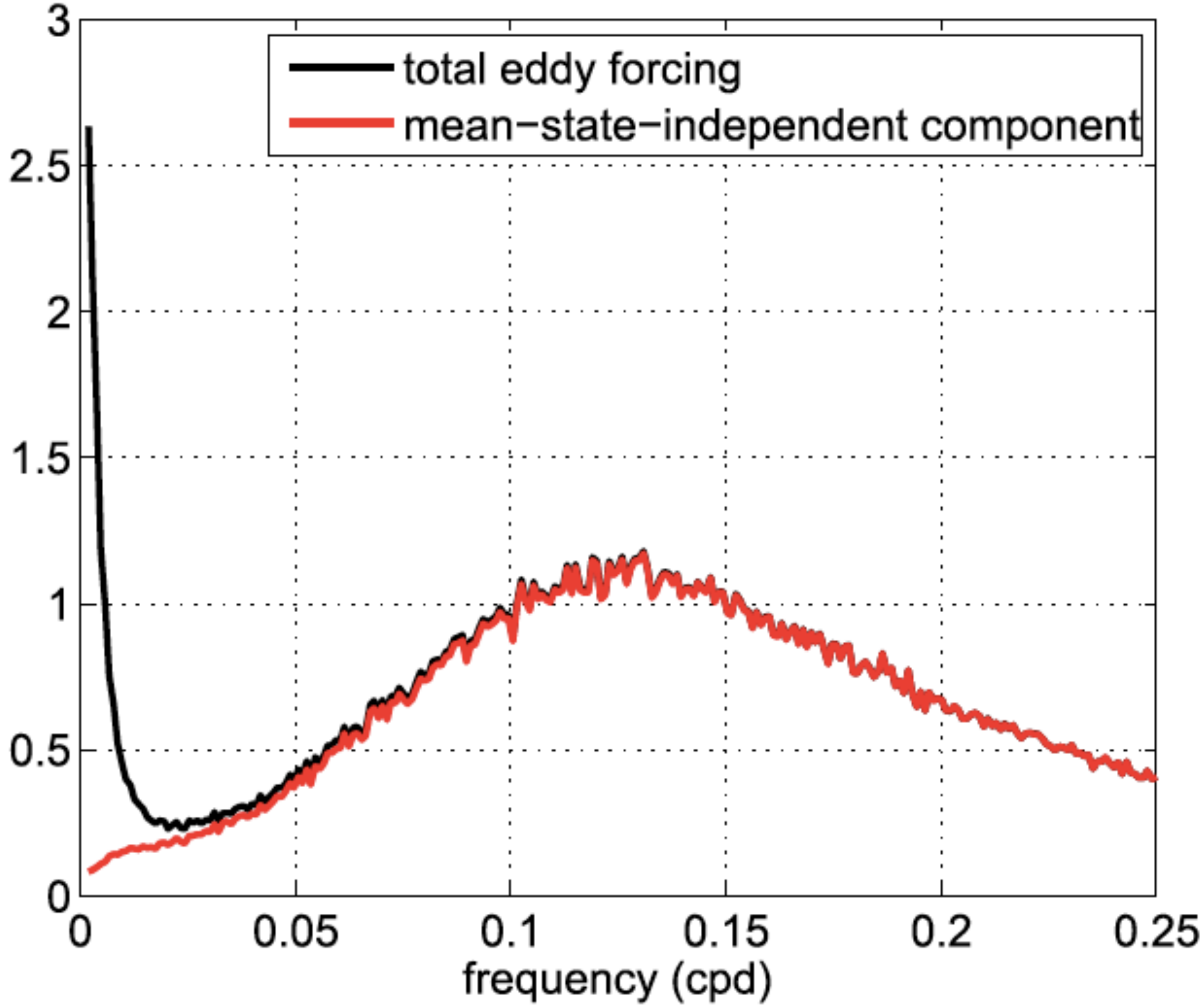
$$0 = \mathbf{M}\vec{X}' + \vec{F}'$$

Estimated feedback strengths



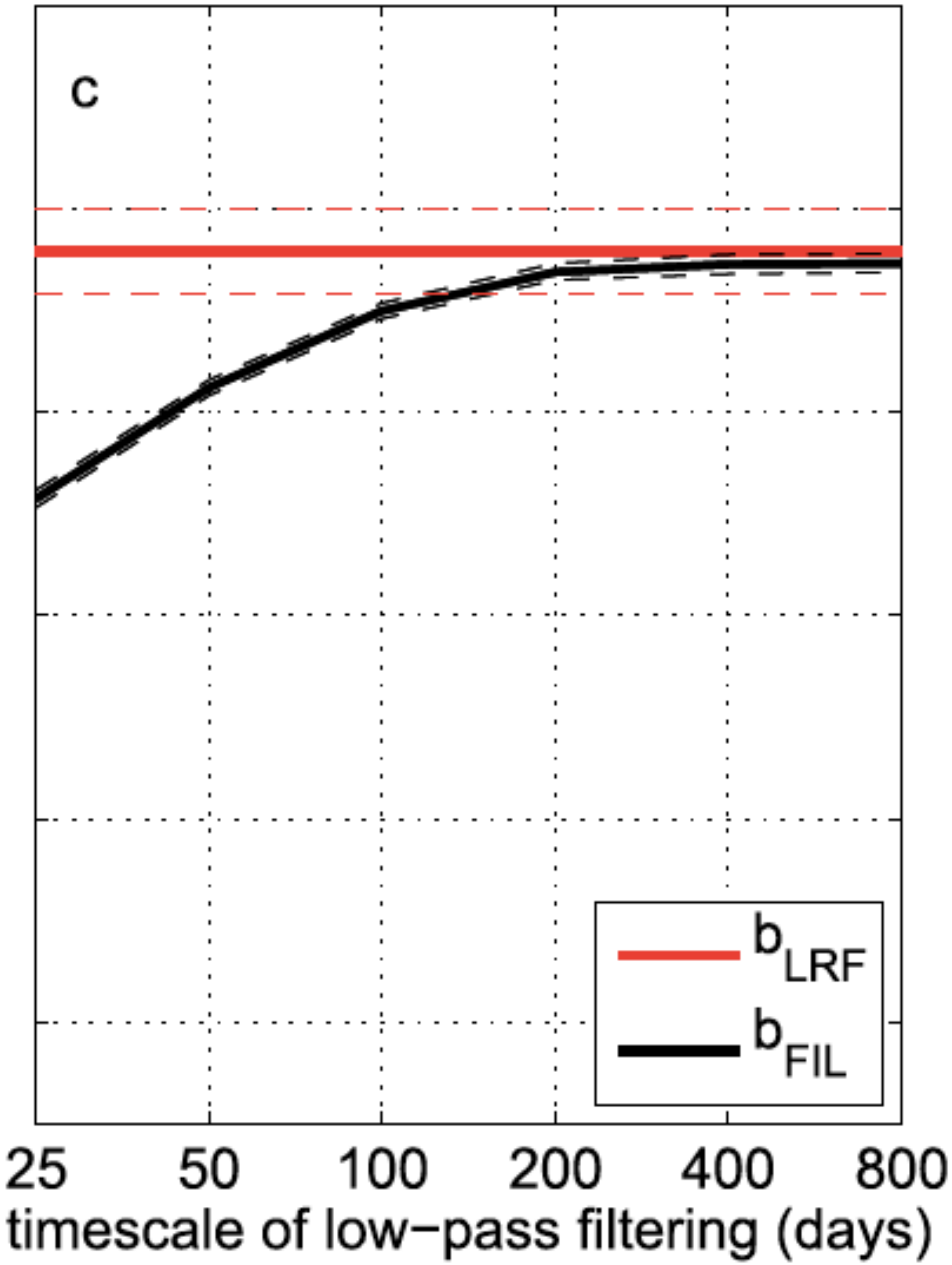
Ma et al., 2017

Power spectrum of eddy forcing

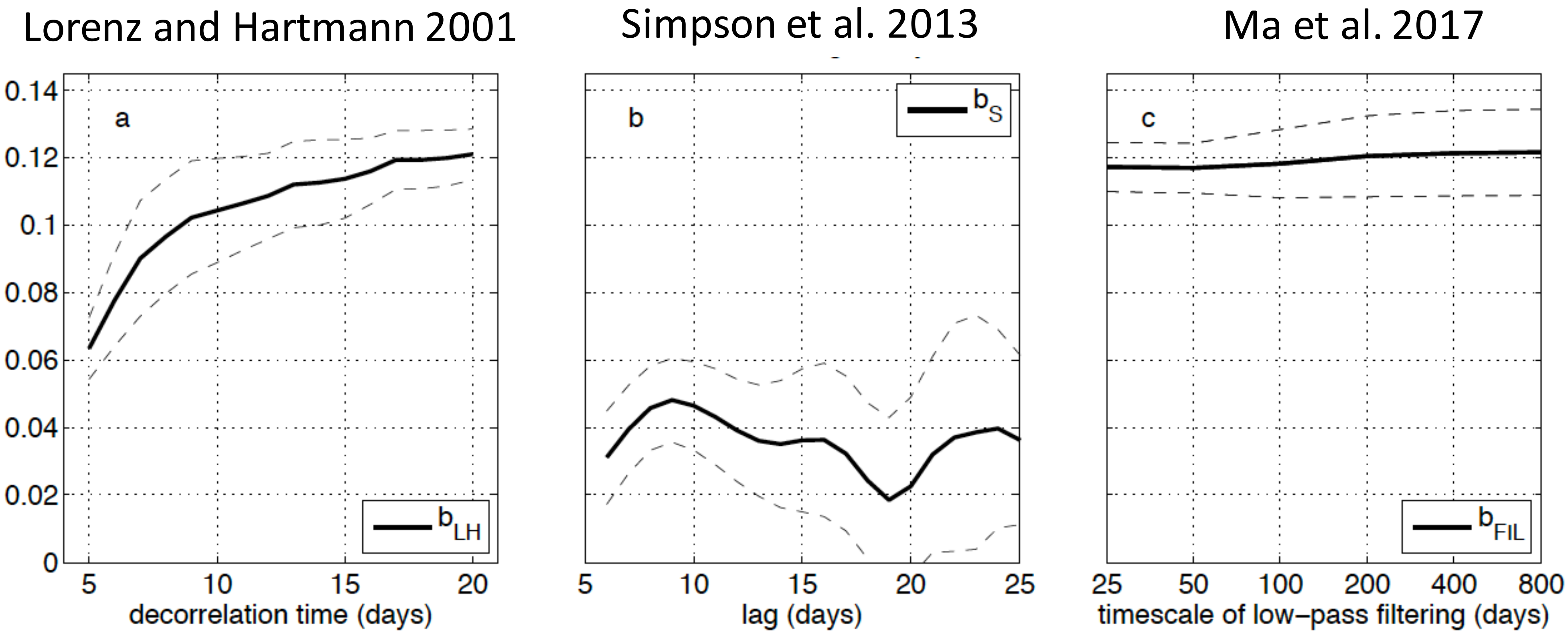


A new low- pass filtering approach (Ma et al., 2017)

Key assumption: The mean-state dependent (or feedback) component dominates the eddy forcing at low frequencies.

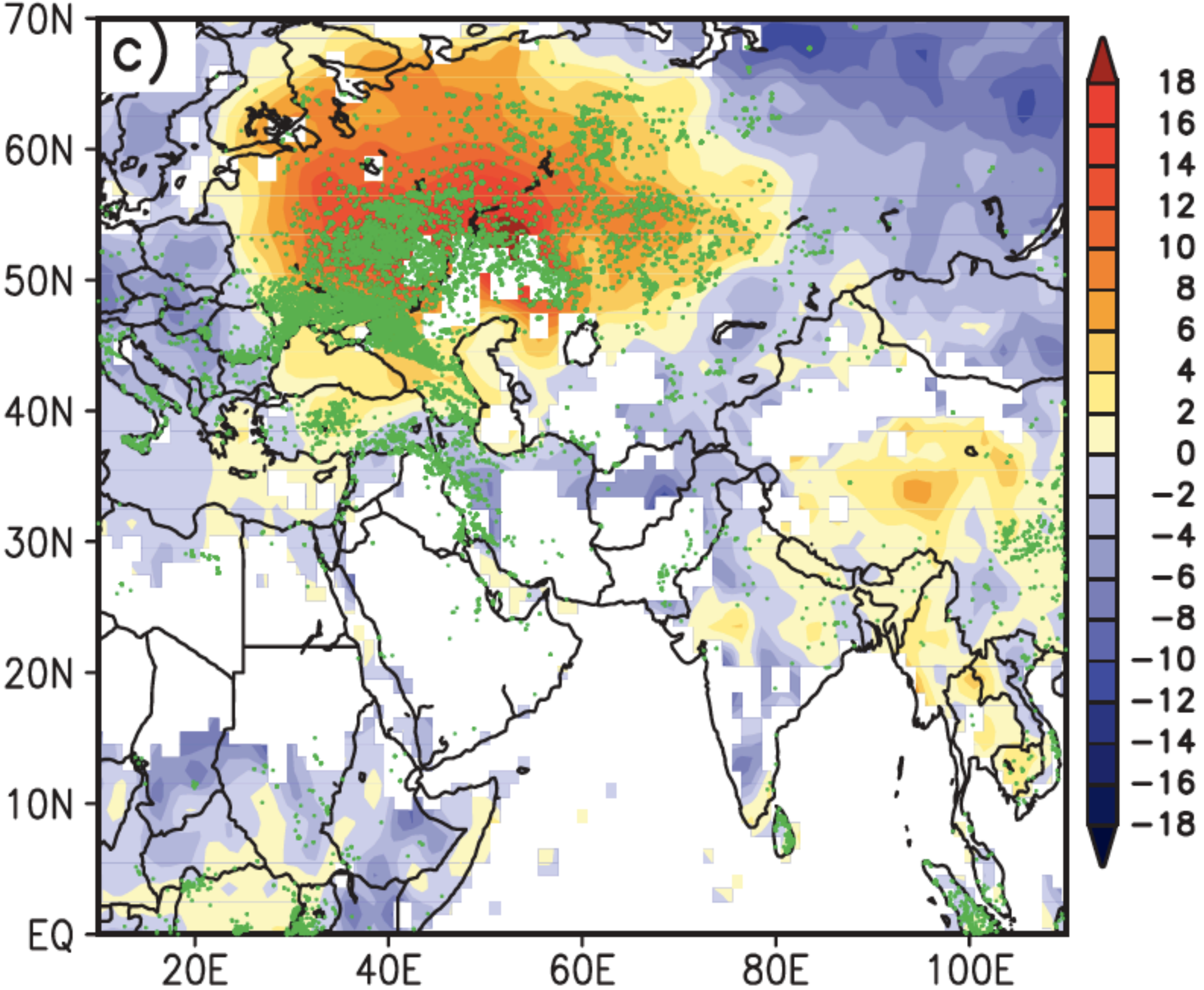


Applied to reanalysis



Further work should include the seasonal cycle

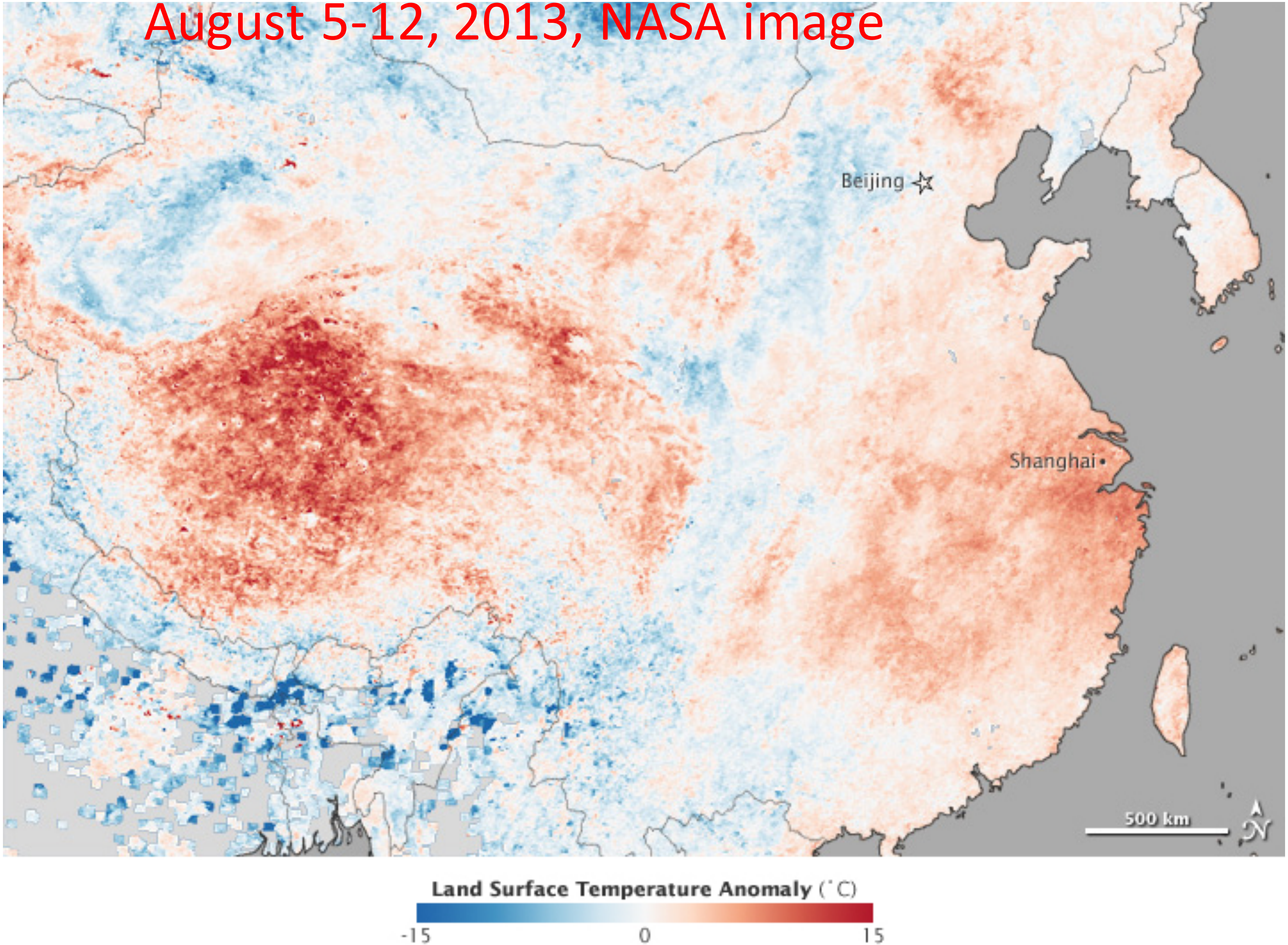
2010 Russian heat wave and Pakistan flood



AIRS temperature anomaly (color)
MODIS daily fire pixels (green dots)

Lau and Kim (2012)

August 5-12, 2013, NASA image



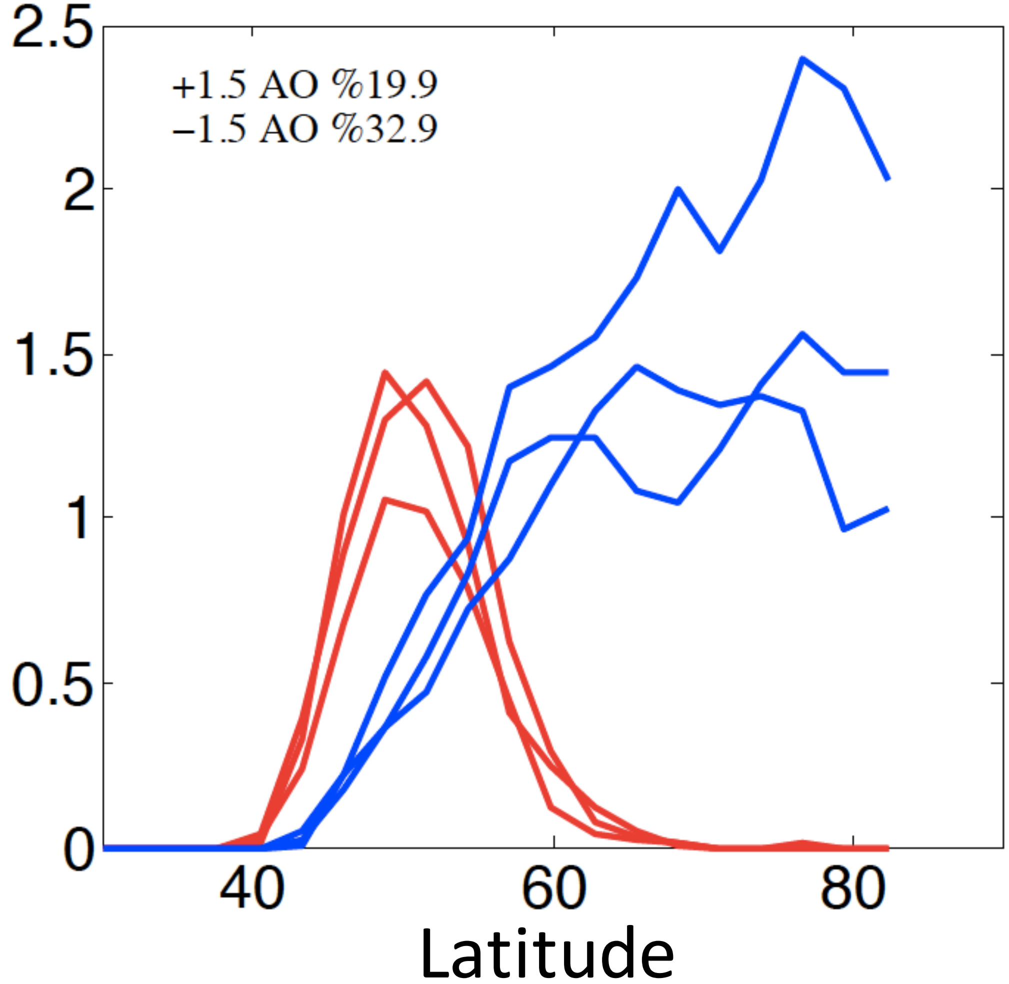
- Arctic amplification reduces the surface equator-to-pole temperature gradient, and hence the jet speed.
- More blocks are observed to happen during the negative phase of the annular mode, which also has a slower jet (Cohen et al. 2014).
- Does it mean there will be more blocks with Arctic amplification (Francis and Vavrus, 2012)?

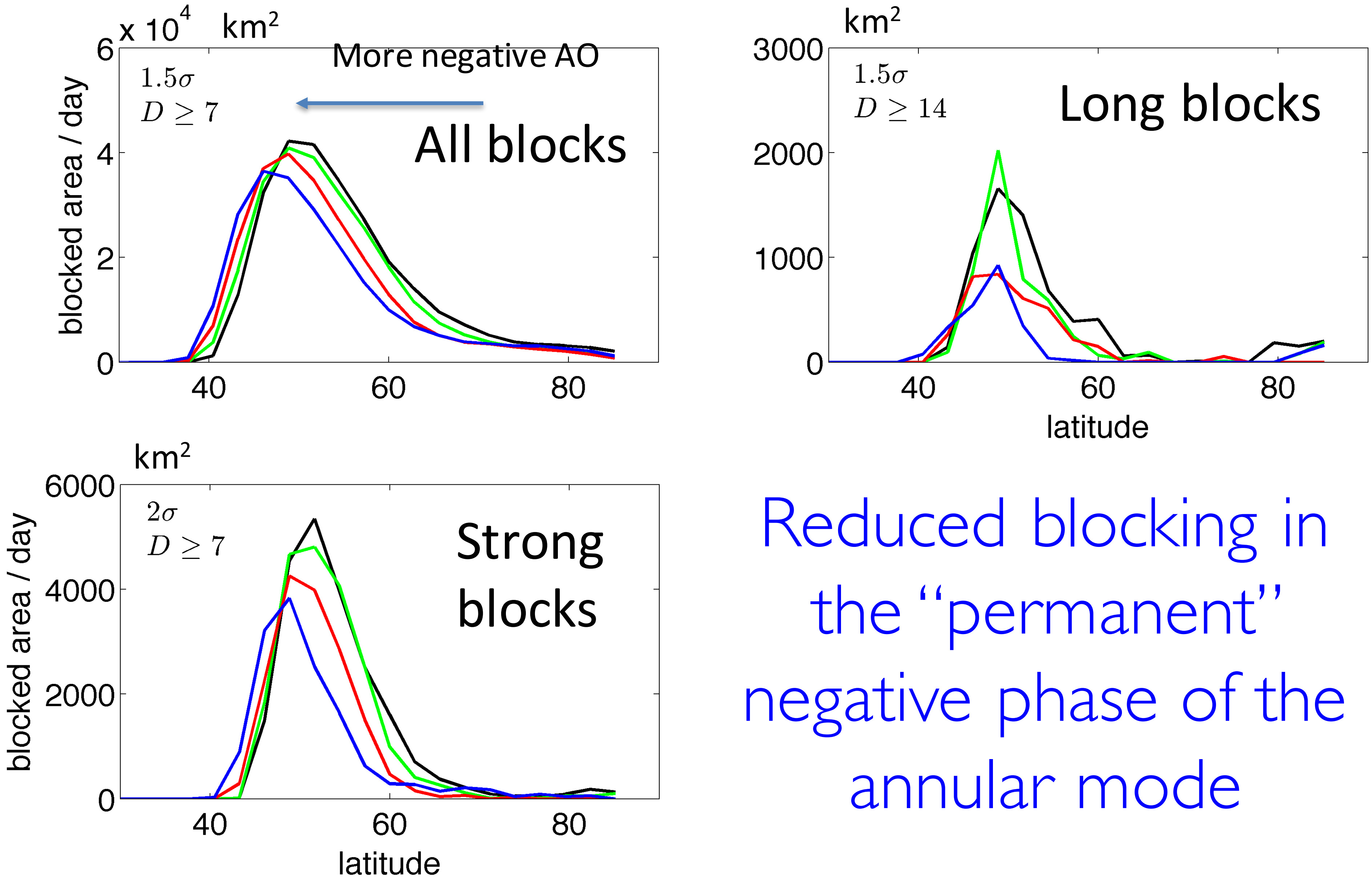
In a simple dry dynamic core, there are also more frequent and more poleward blocks in the negative phase of the annular mode (when the jet is weaker and more equatorward)

Y axis is % per latitude bin (2.8 degrees)

Positive phase

Negative phase

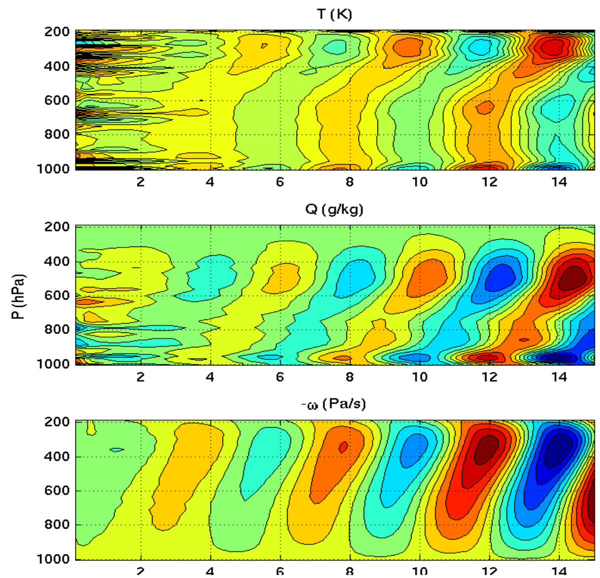


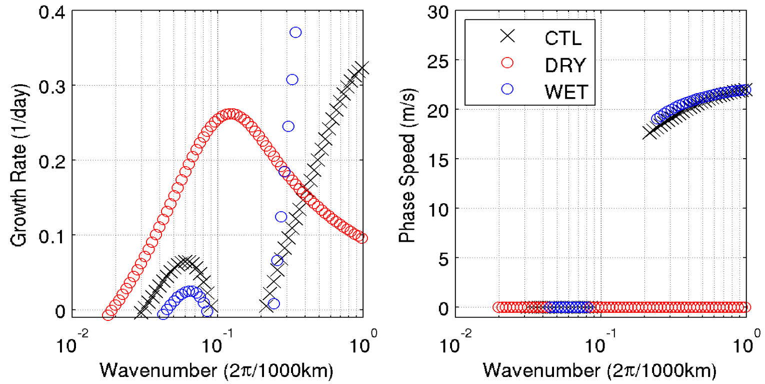


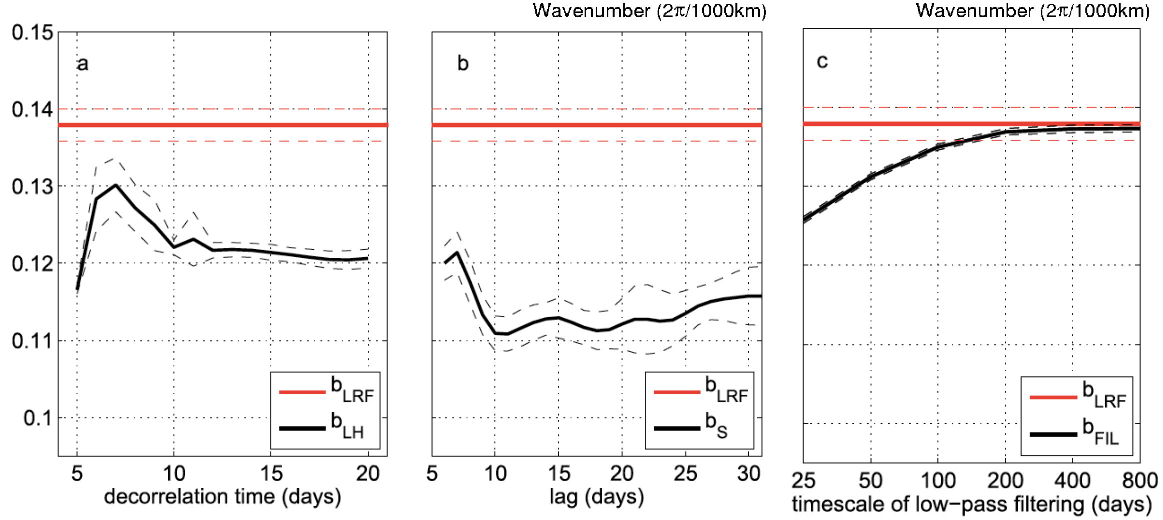
Reduced blocking in the "permanent" negative phase of the annular mode

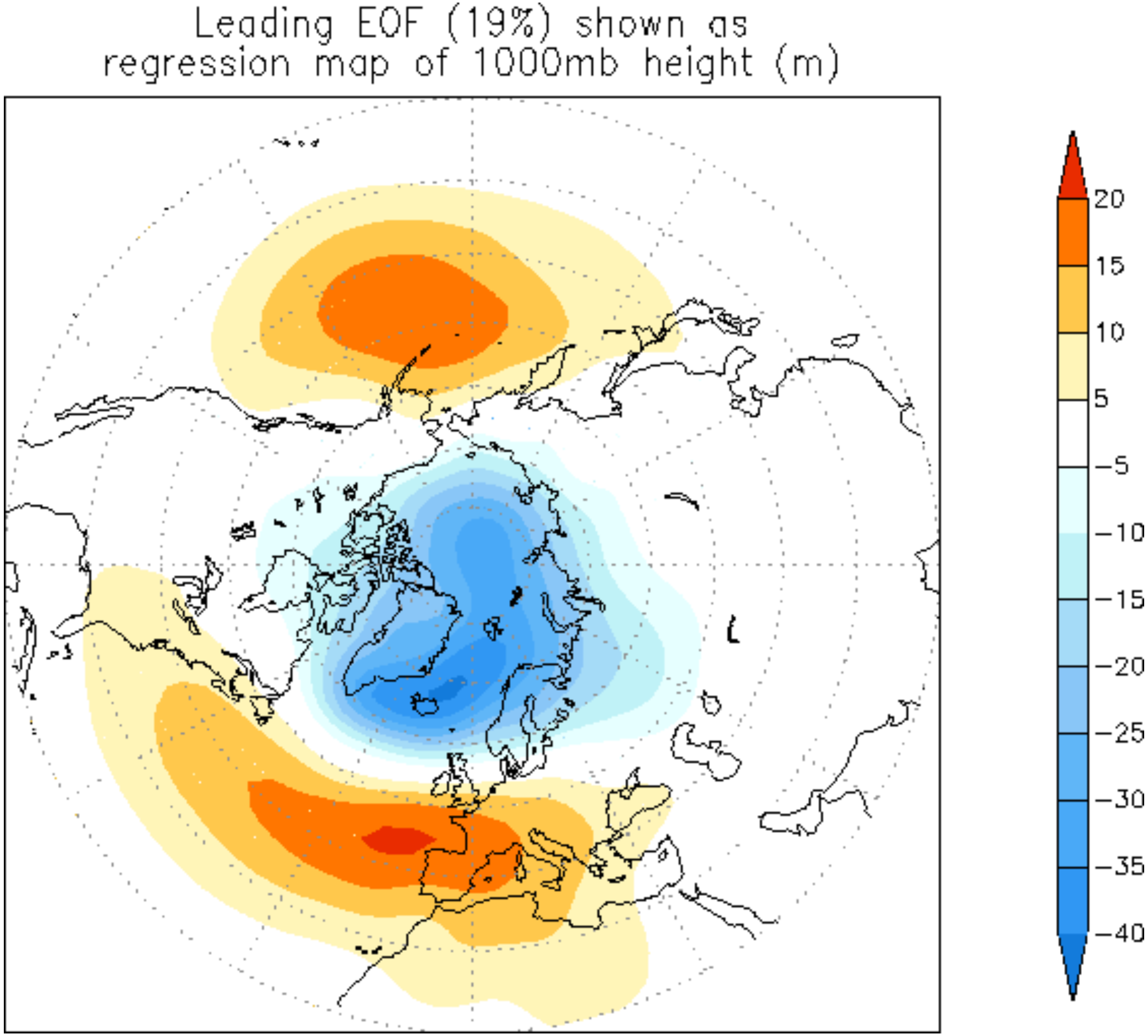
Summary

Linear response functions can be usefully constructed and applied in a number of problems in atmospheric dynamics.





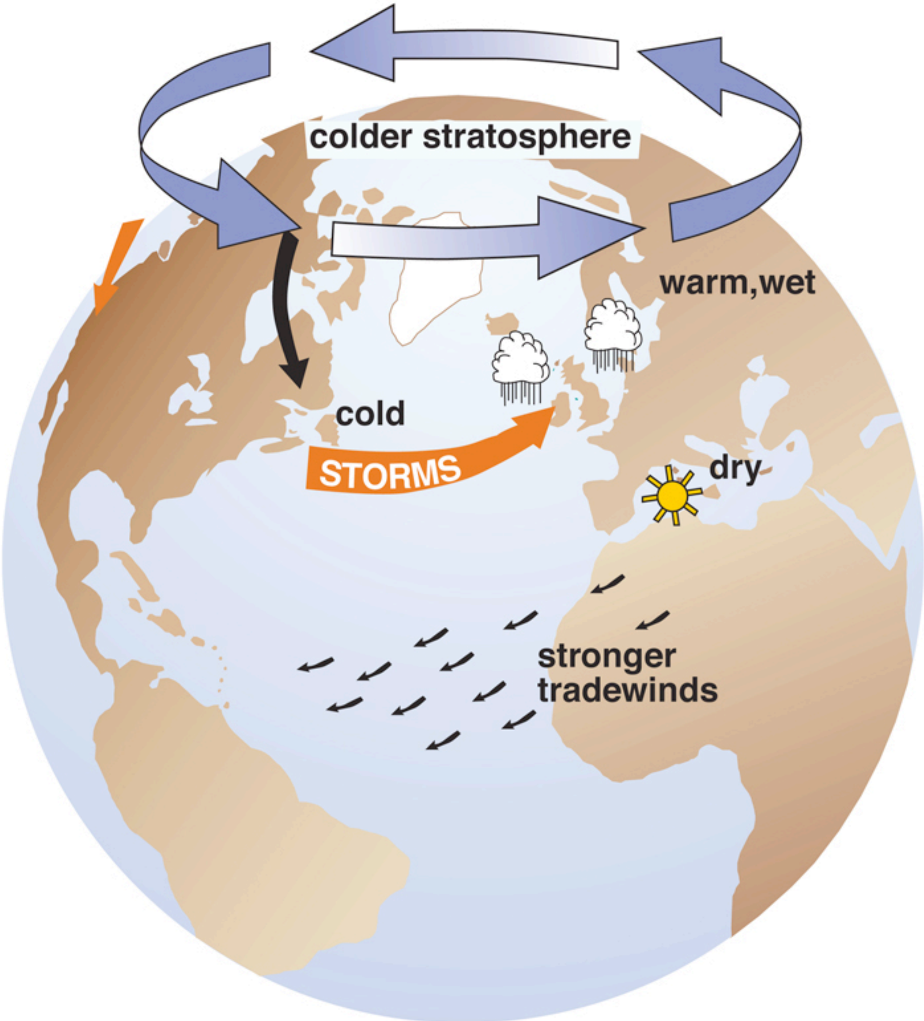


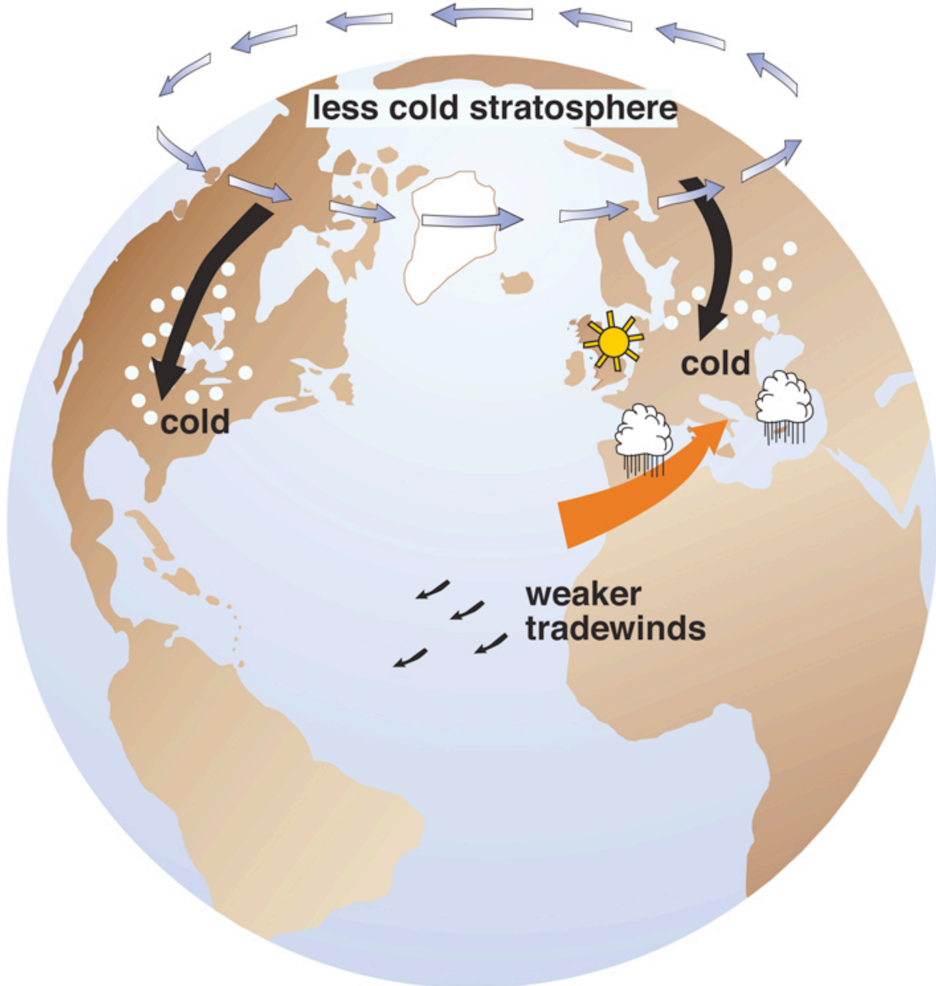


Northern annular mode

Credit: NCEP CPC
Following Thompson and Wallace, 2000

Northern annular mode

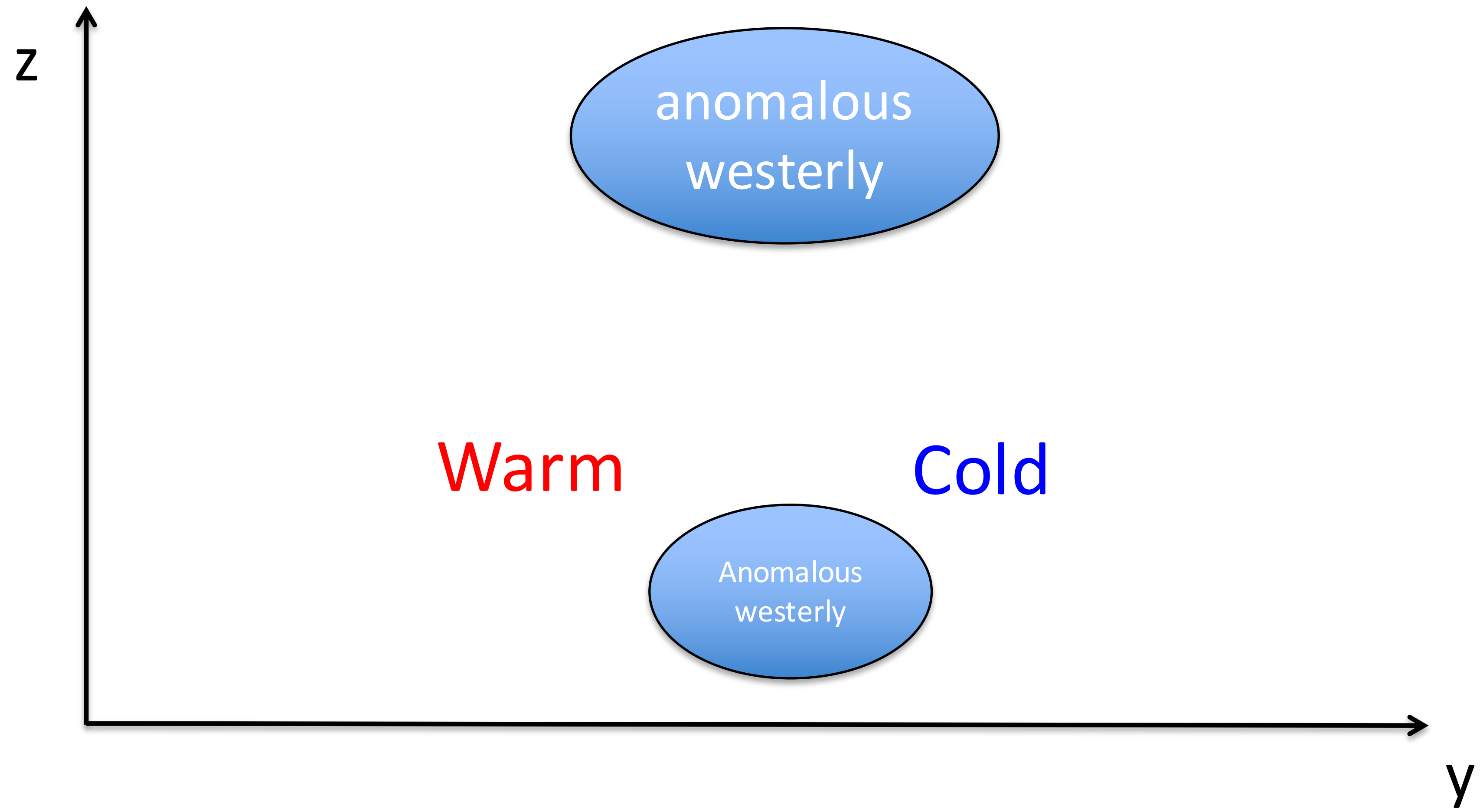




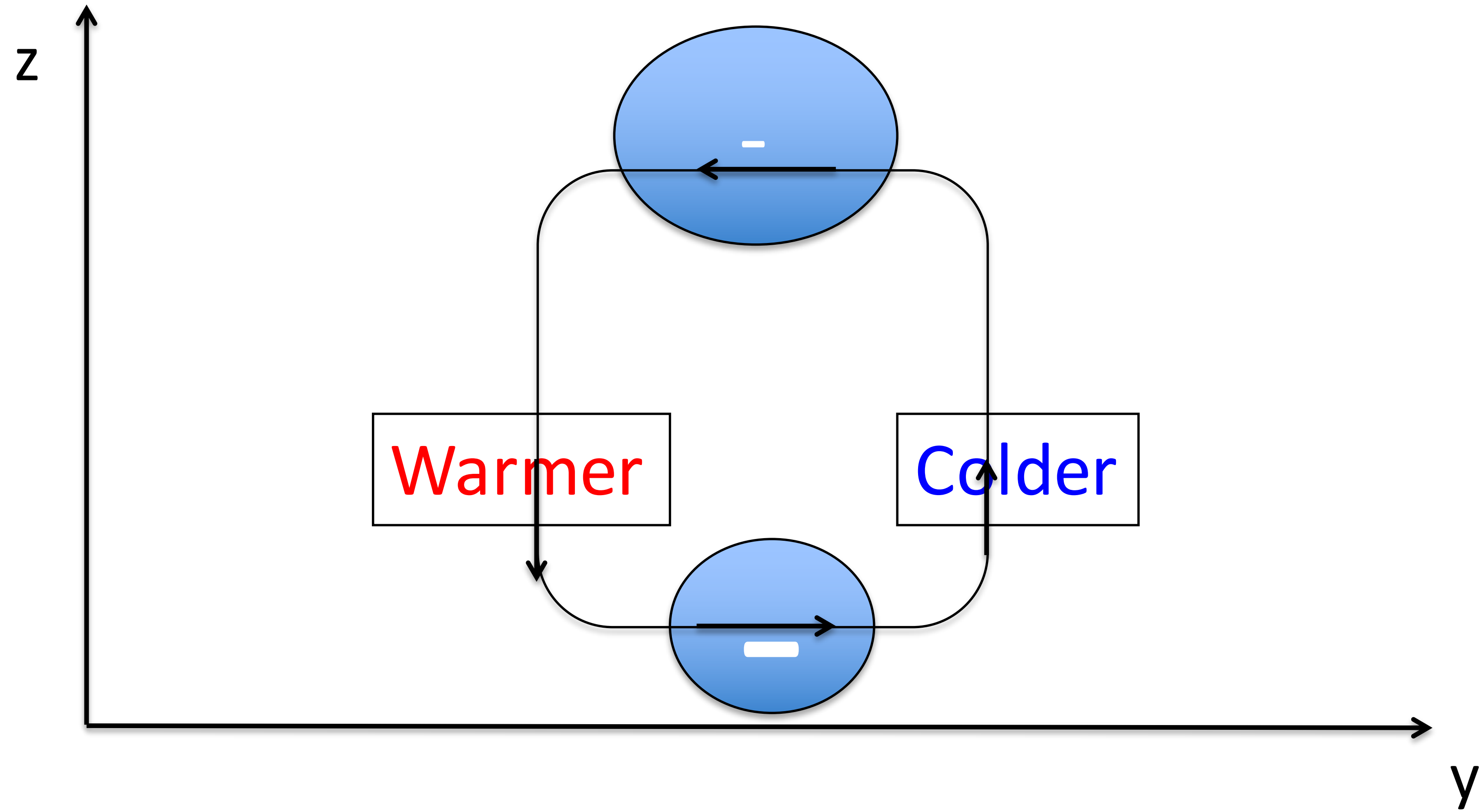
From Mike Wallace

- Why do convectively coupled waves exist?
- What set their scales and speeds?
- Why are certain wave types stronger than the others?

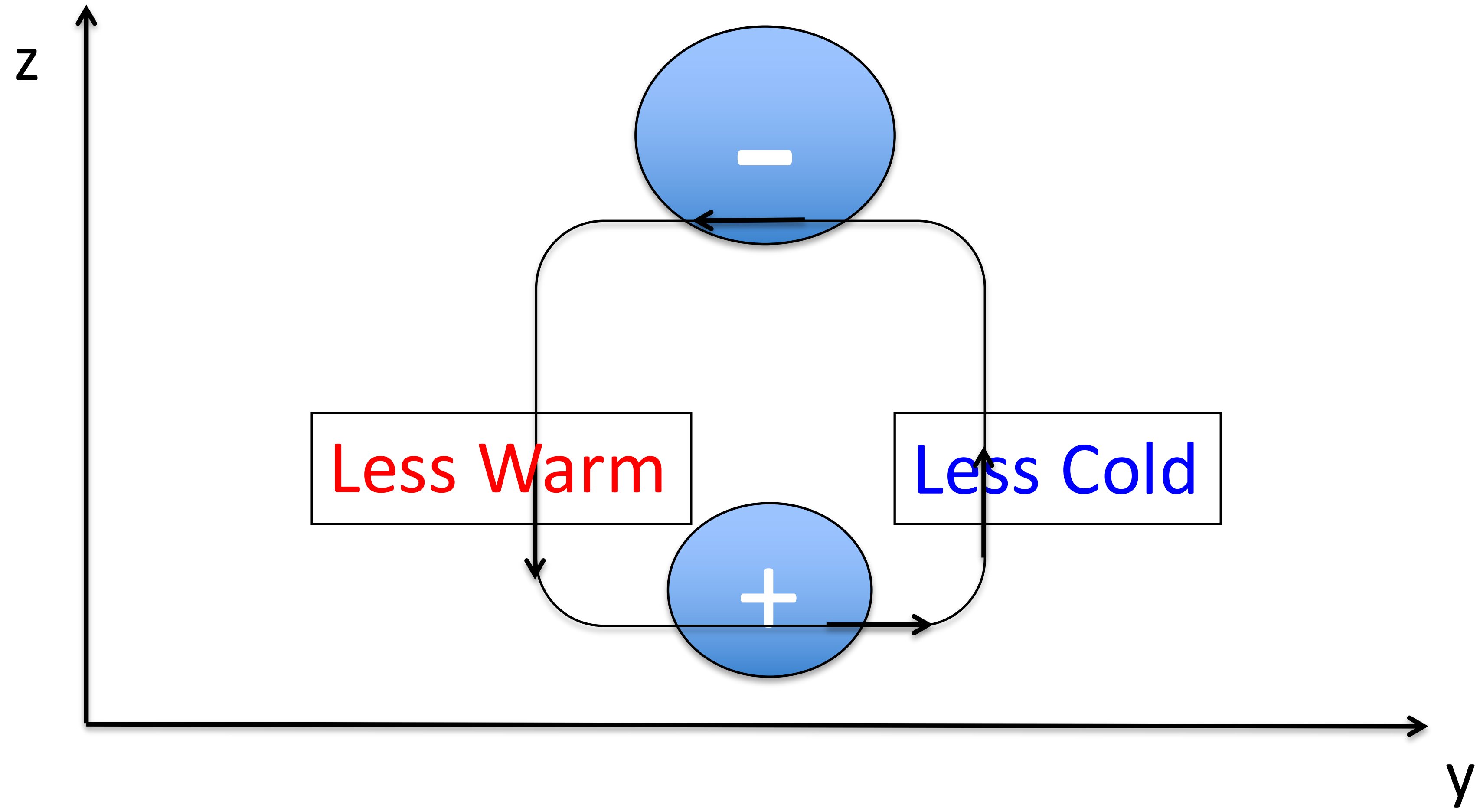
Let’s start with a jet anomaly



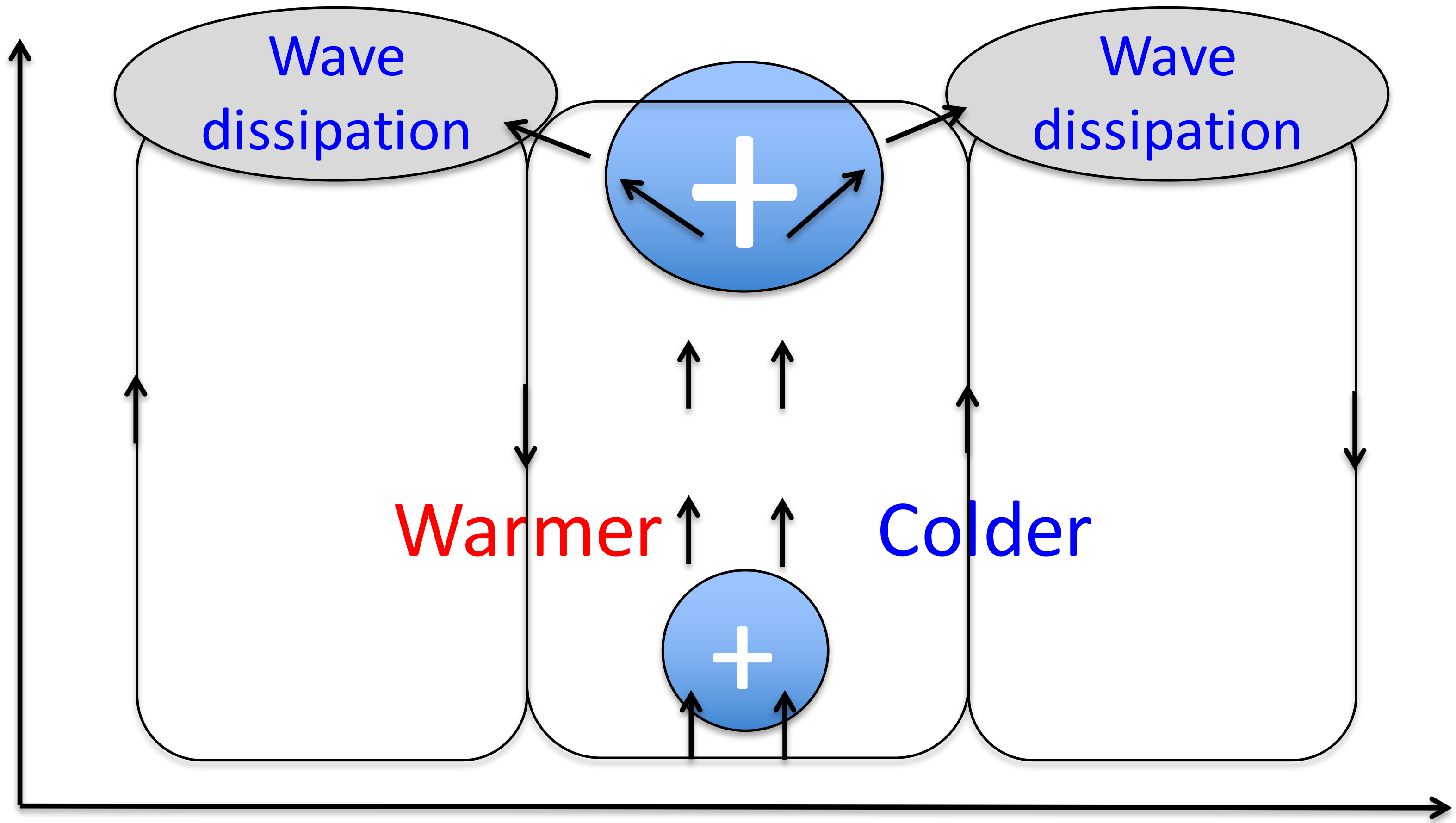
Boundary layer friction reduces boundary layer winds and enhances temperature gradient



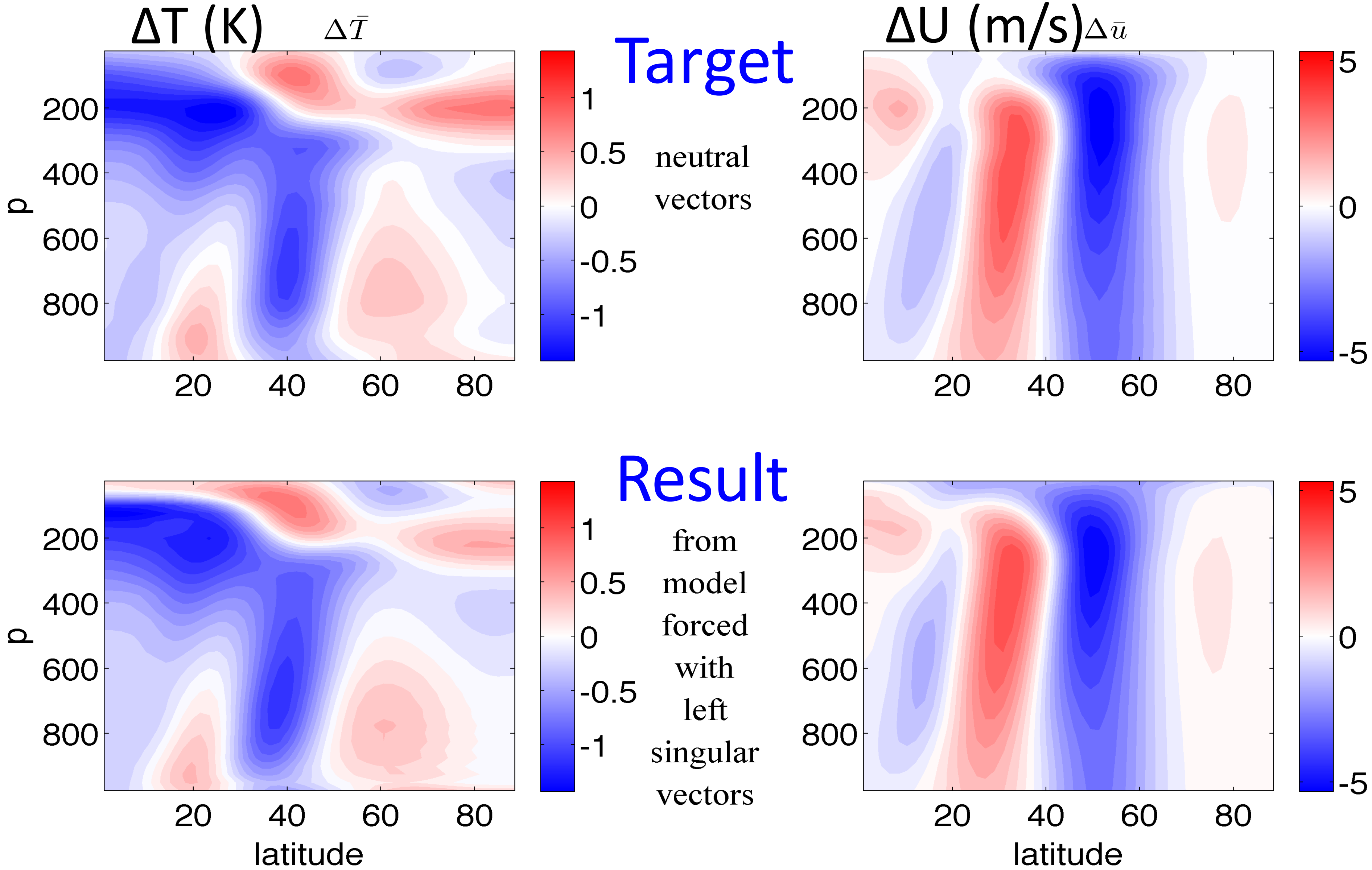
Eddy heat flux reduces temperature gradient and enhances boundary layer winds and reduces upper level winds



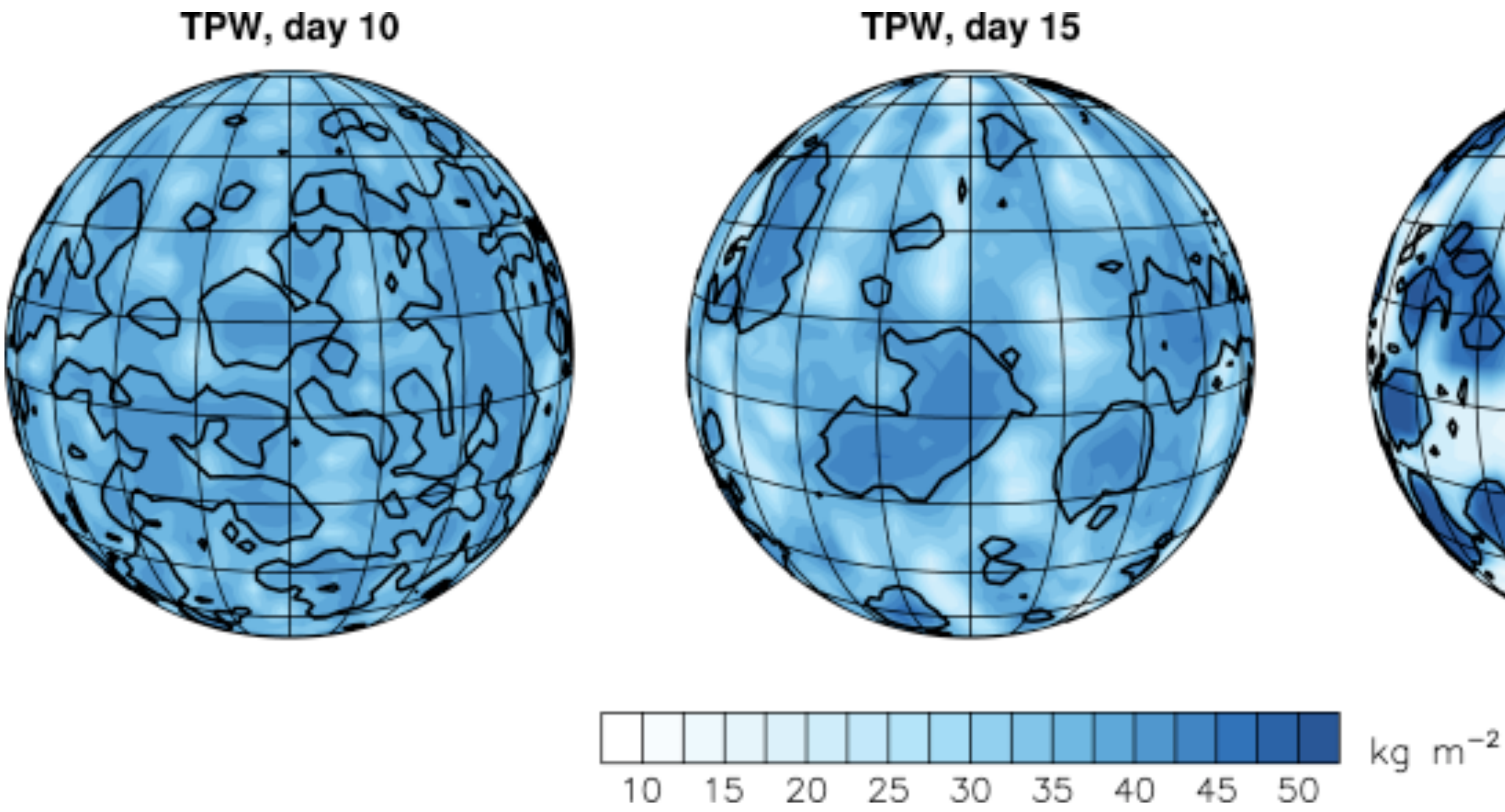
Eddy momentum flux enhances upper level winds and the temperature gradient



Shooting for a "permanent" negative phase of the annular mode

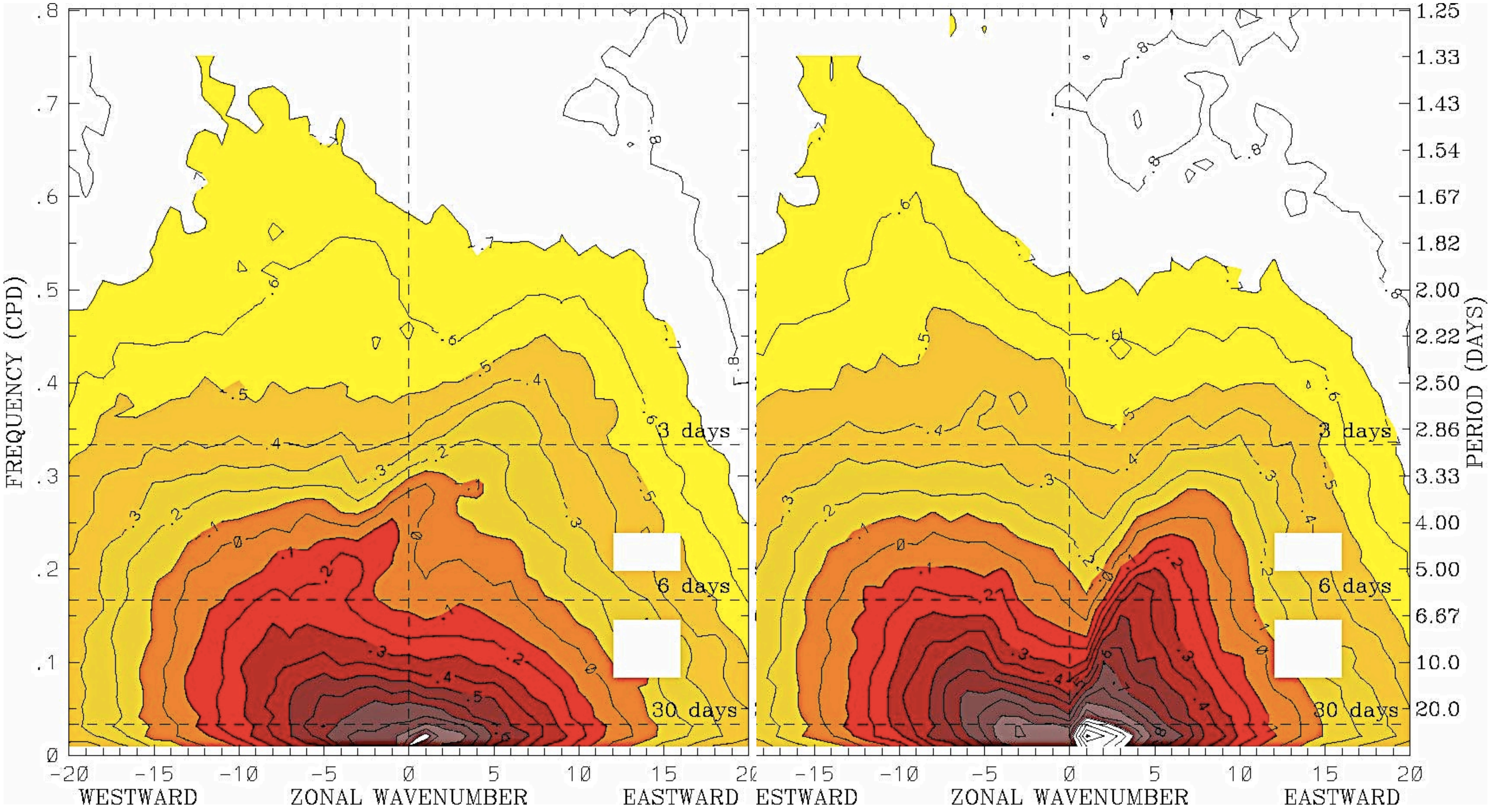


Self-aggregation with a globally uniform SST and no rotation



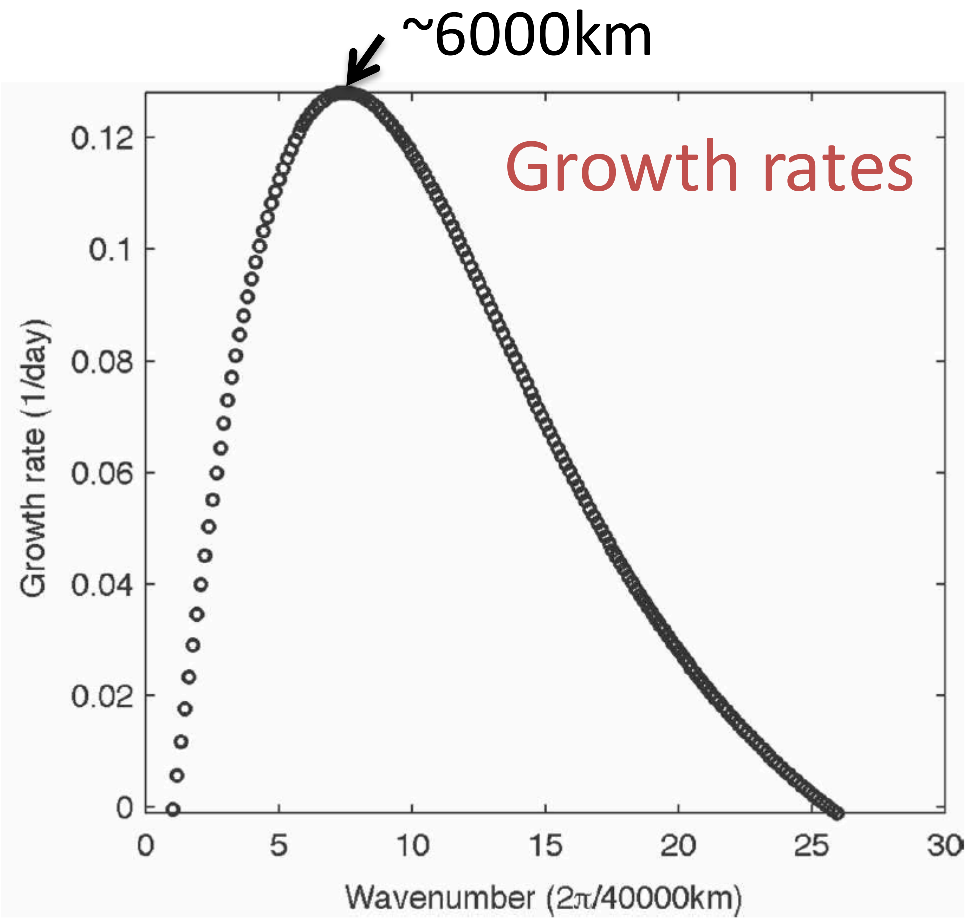
Arnold and Randall, 2015

Space-time spectra of OLR (log_{10}Power)
(averaged over 15N-15S, 20 years)



Wheeler and Kiladis, J. Atmos. Sci., 1999

To identify the basic instability mechanisms, we constructed a simple model (6 to 2 ODEs) that is consistent with the linear response functions



Kuang, 2008

But, there are also stabilizing processes:

Through enhanced detrainment of updrafts, convection will damp the dry anomaly, with a timescale about 1-2 days in Radiative Convective Equilibrium.

Method of construction

$$\left[\left(\frac{d\vec{x}}{dt}\right)_1 \quad \left(\frac{d\vec{x}}{dt}\right)_2 \quad \cdots \quad \left(\frac{d\vec{x}}{dt}\right)_n\right] = M\left[\vec{x}_1 \quad \vec{x}_2 \quad \cdots \quad \vec{x}_n\right]$$

(minus) prescribed forcing
(precisely known)

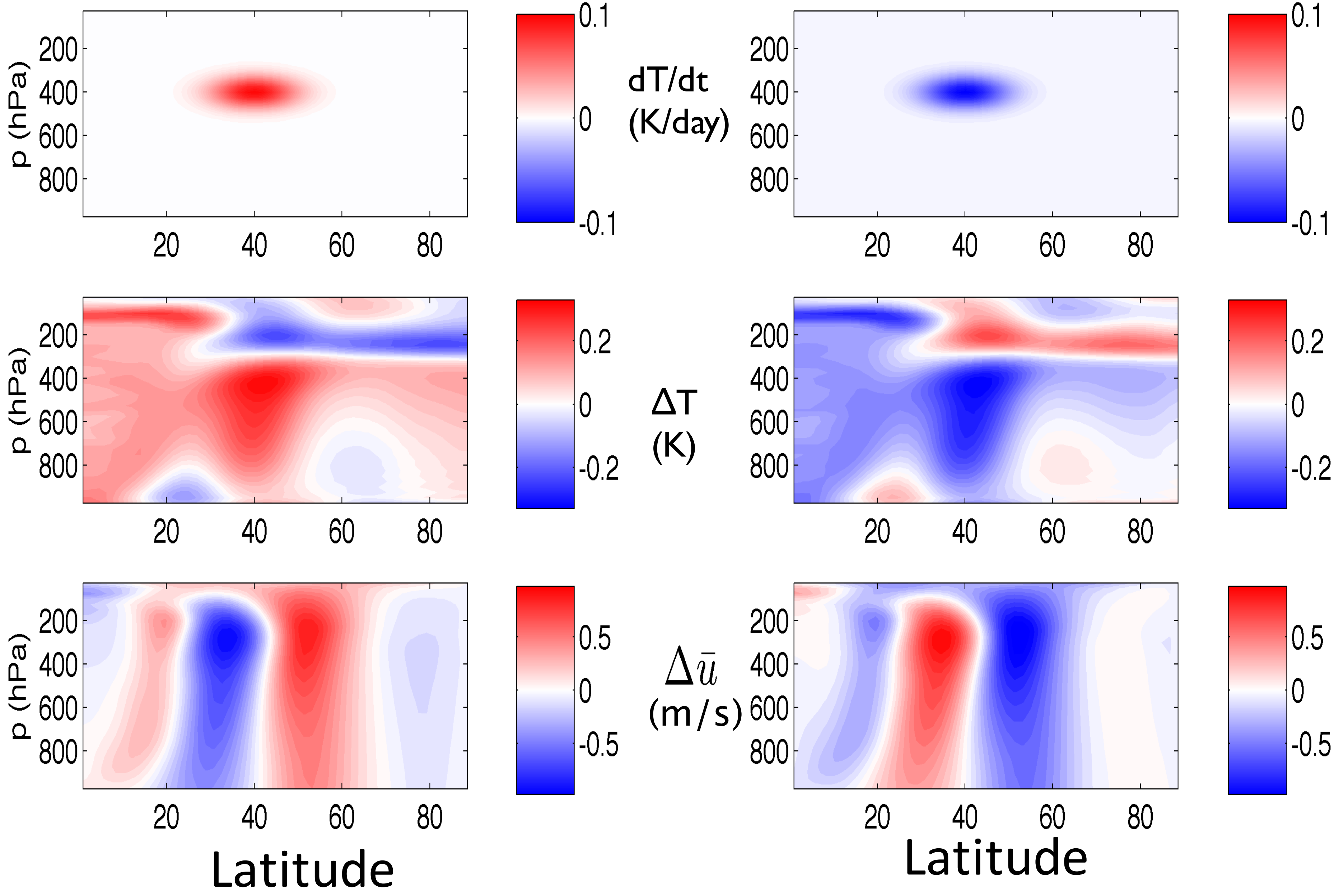
Equilibrium response X
(has uncertainties)

Errors in eigenvalue λ: $|\delta\lambda| \propto |\lambda^2| \|\delta X\|$

- The fastest decaying modes of M (i.e. with the largest (in modulus) eigenvalues) have the largest errors
- The slowest decaying modes of M (i.e. the smallest eigenvalues) are the most accurate.
- The latter are of the most interest for large-scale flows

Analogous to what's done for moist convection in Kuang (2010)

An example



p (hPa)
200
400
600
800
20
40
60
80
0.1
0
-0.1
dT/dt
(K/day)
ΔT
(K)
0.2
-0.2
$\Delta\bar{u}$
(m/s)
0.5
-0.5
Latitude
Latitude

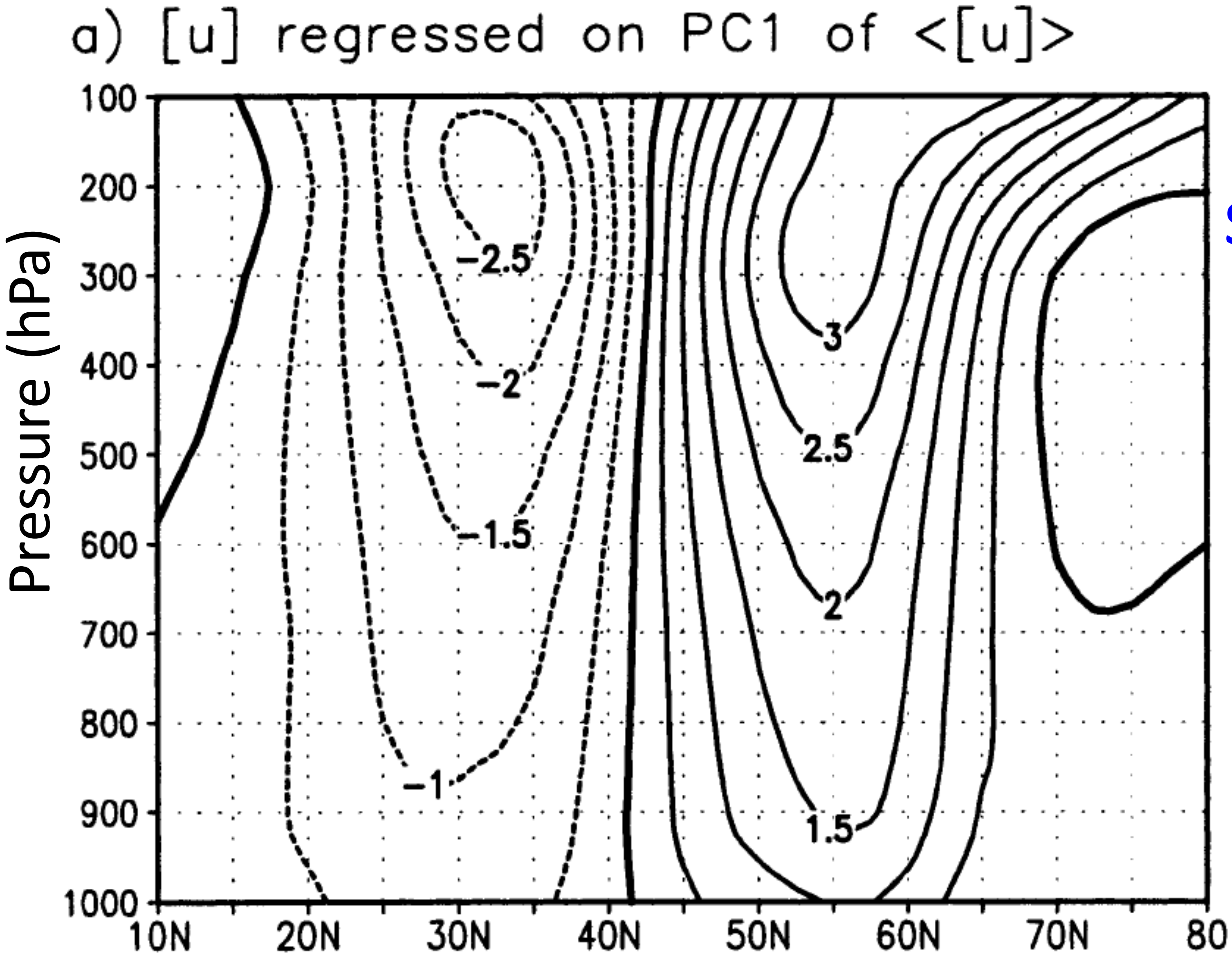
Jet stream variability

- Annular modes (leading mode of internal variability as well as of response to external forcing)
- Blocks (contribute to extreme weather such as heat waves, cold spells, droughts, and heavy precipitation)

The set of forced runs provides a mapping between time tendencies and the state vector.

The linear response functions are linear combinations of the forced runs so that the state vectors, instead of the tendencies, are compact in the mapping.

Eddy statistics from this set of forced runs can be linearly combined in a similar fashion to give changes in eddies caused by a particular change in the state vector

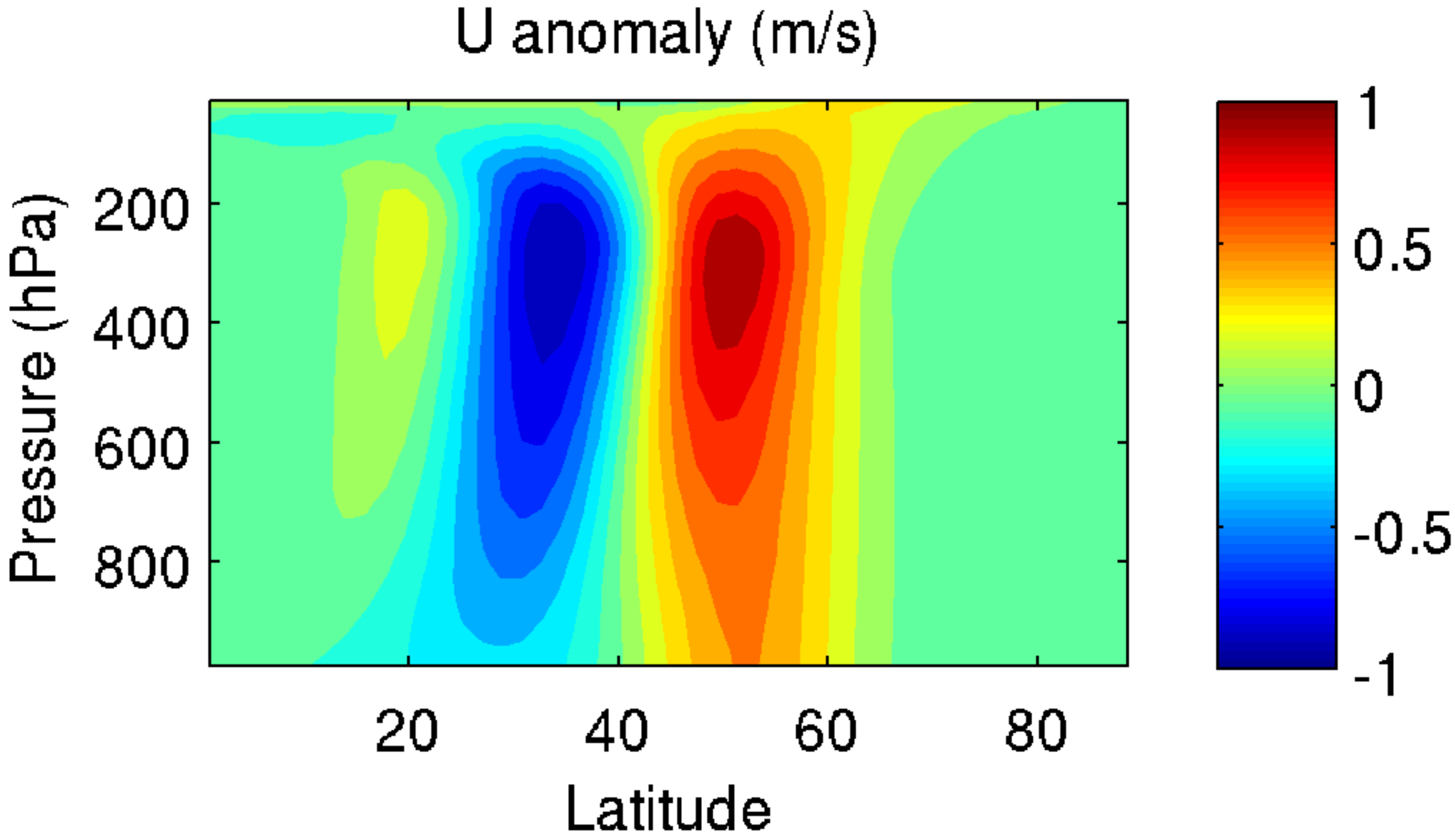


Vertical structure in zonal wind

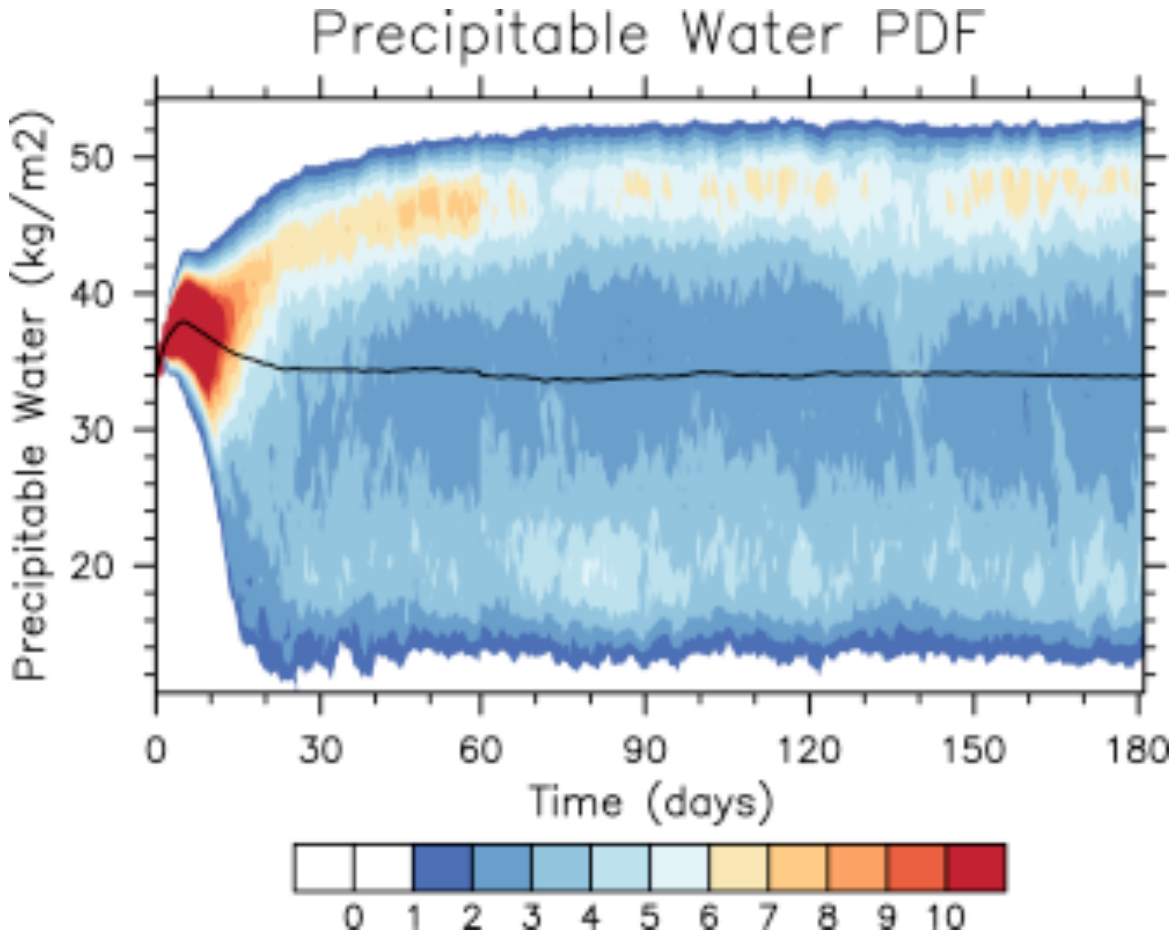
Lorenz and Hartman 2003

Annular mode in the simple model

(First principle component of the control run daily data)

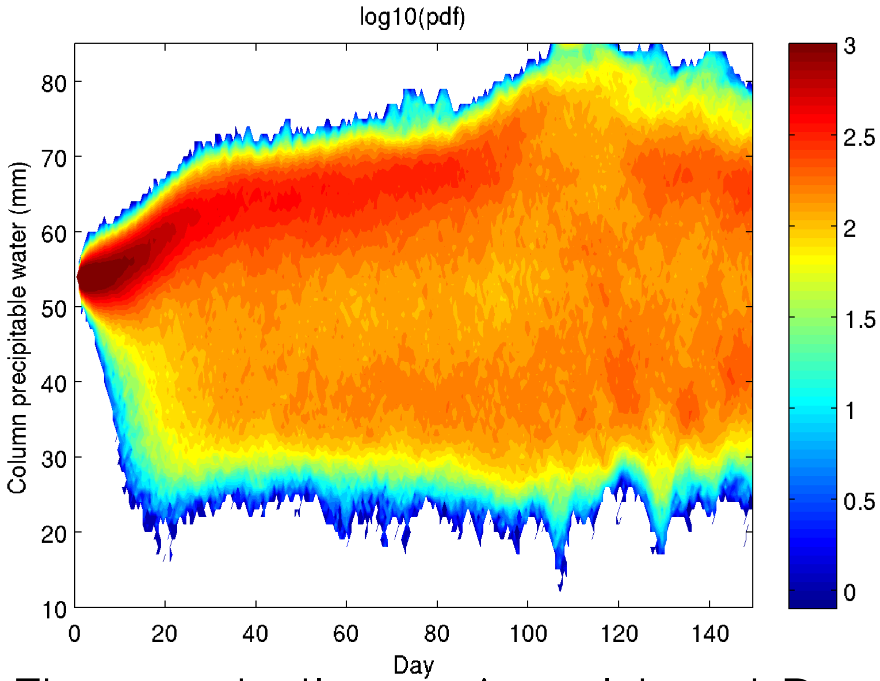


Time evolution of the PDF

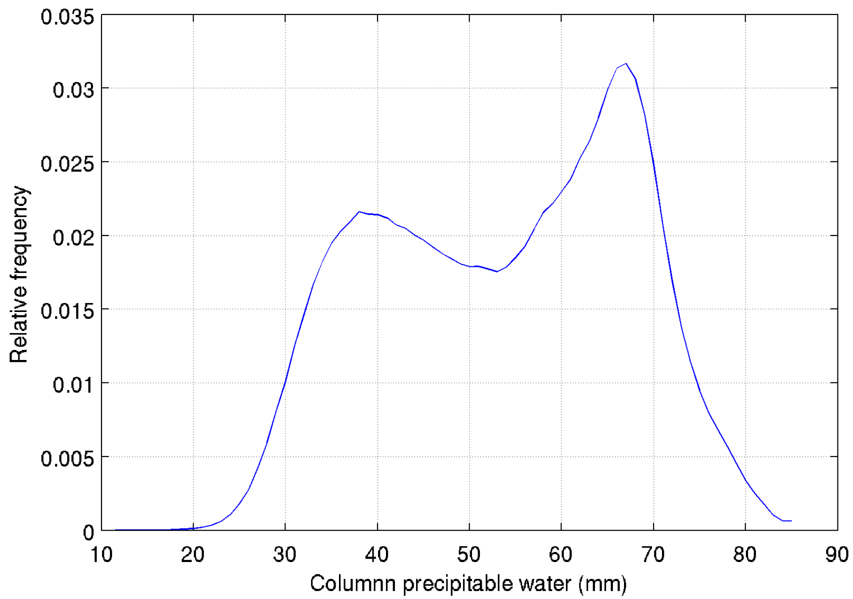


- This self-aggregation in SPCAM has a column moist static energy (MSE) budget similar to those in cloud-resolving models (e.g. Wing and Emanuel, 2013)
- Self aggregation in this simple setting is intriguing and potentially relevant to the MJO

Time evolution of the PDF



PDF averaged over days 50-150



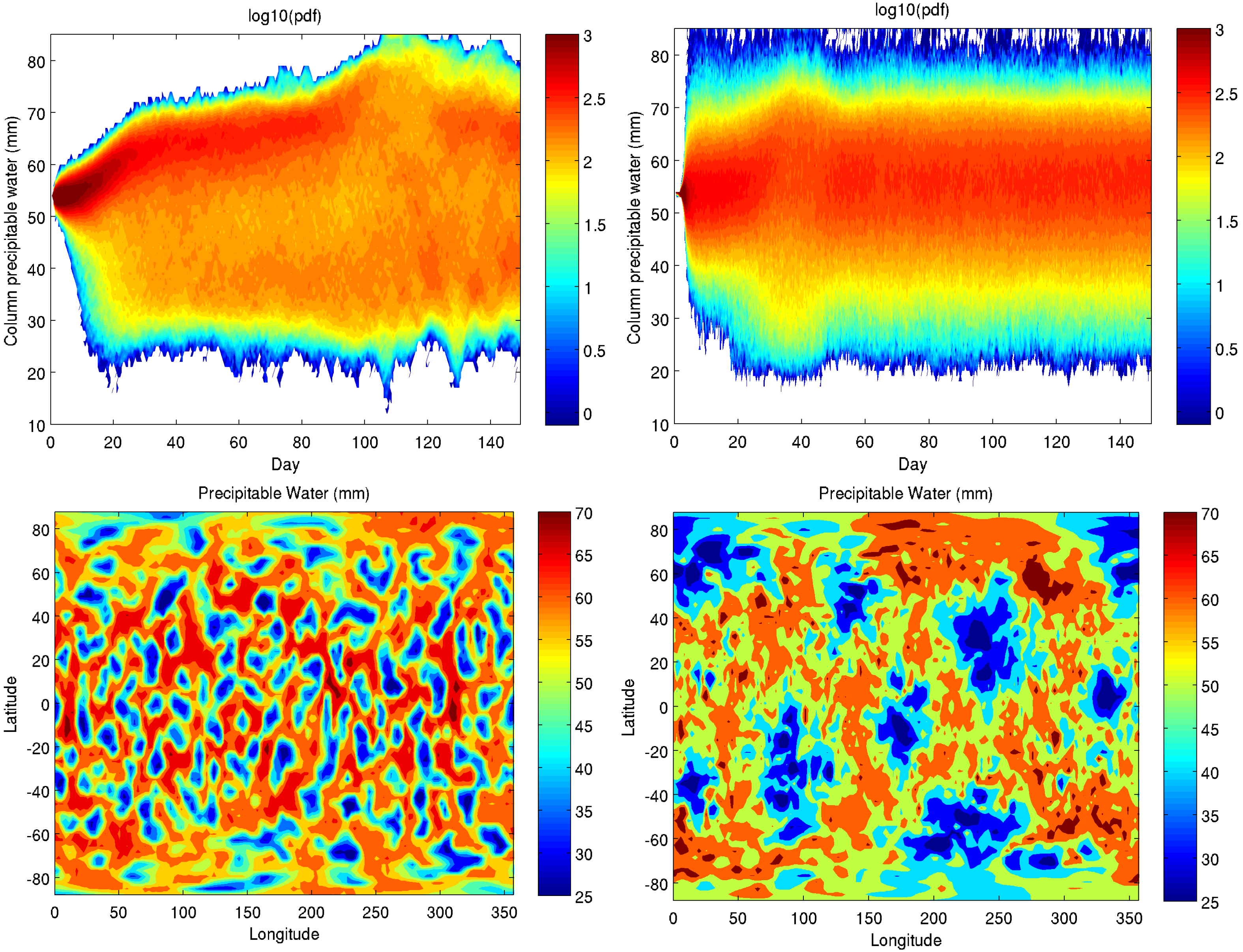
- Figures similar to Arnold and Randall (2015), which further showed that this self-aggregation in SPCAM has a column moist static energy (MSE) budget similar to those in cloud-resolving models (e.g. Wing and Emanuel, 2013)
- Self aggregation in this simple setting is intriguing and potentially relevant to the MJO

Replace the CRM in SPCAM with its linear response function

Cloud resolving model | Linear response function

Time evolution of precipitable water PDF

Snapshot of precipitable water at day 30



Both with bulk formula for surface fluxes with constant 5m/s wind

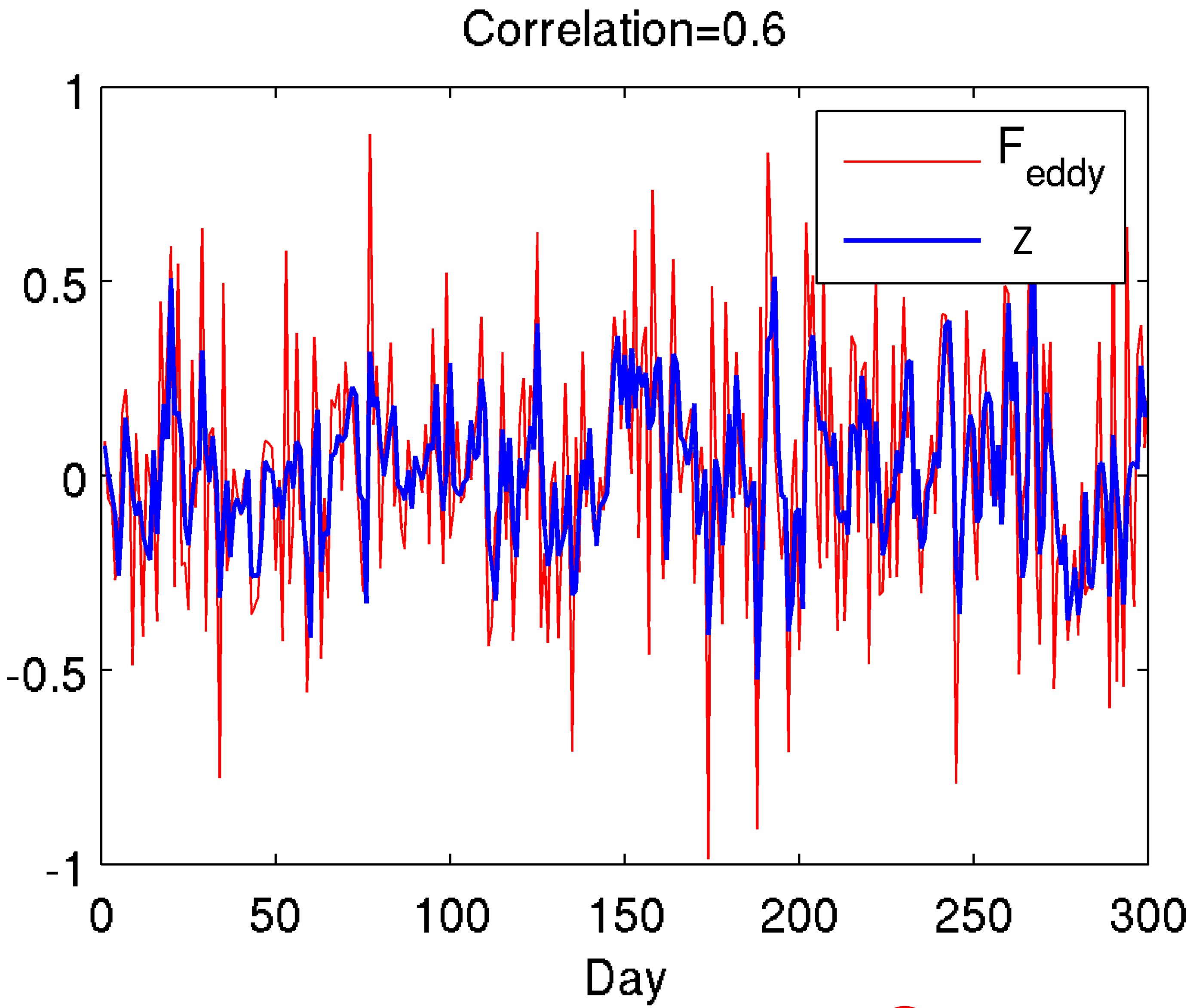
Let's look at a simple example

$$\frac{dz}{dt} = -\frac{z}{\tau} + F_{eddy}$$

$$F_{eddy} = bz + \xi$$

$$\tau = \ 1 \ \ day$$

Random noise independent of z

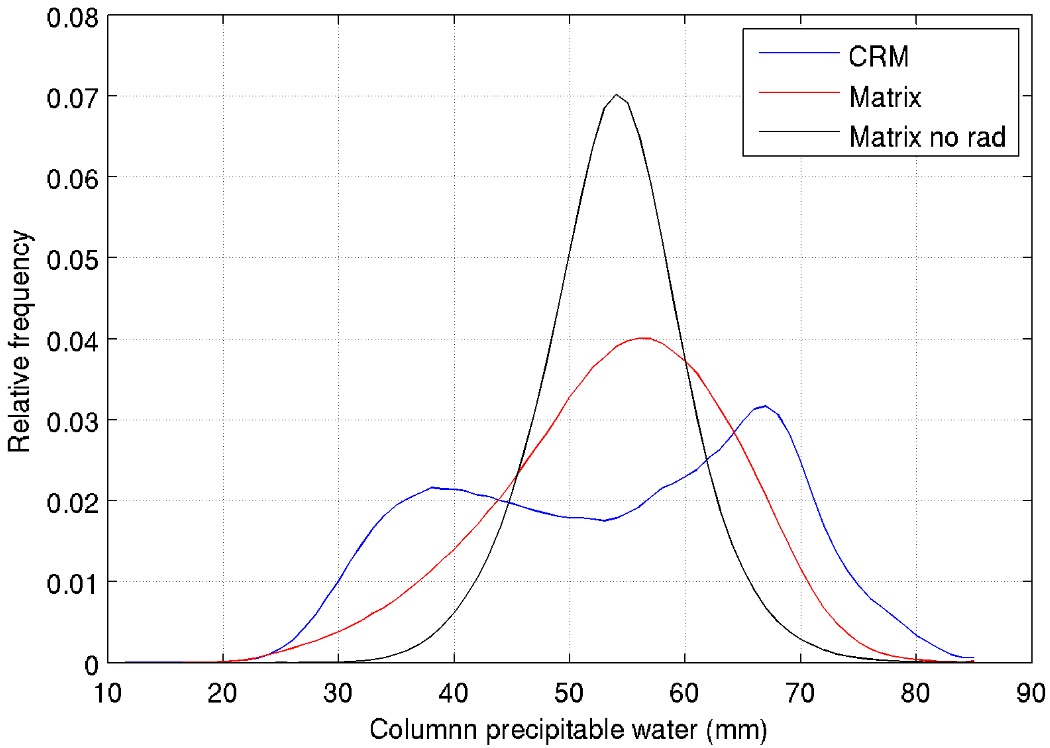


$$\frac{dz}{dt} = -\frac{z}{\tau} + F_{eddy}$$

$$F_{eddy} = bz + \xi$$

$$\tau = \ 1 \ \ day$$

Generated with b=0

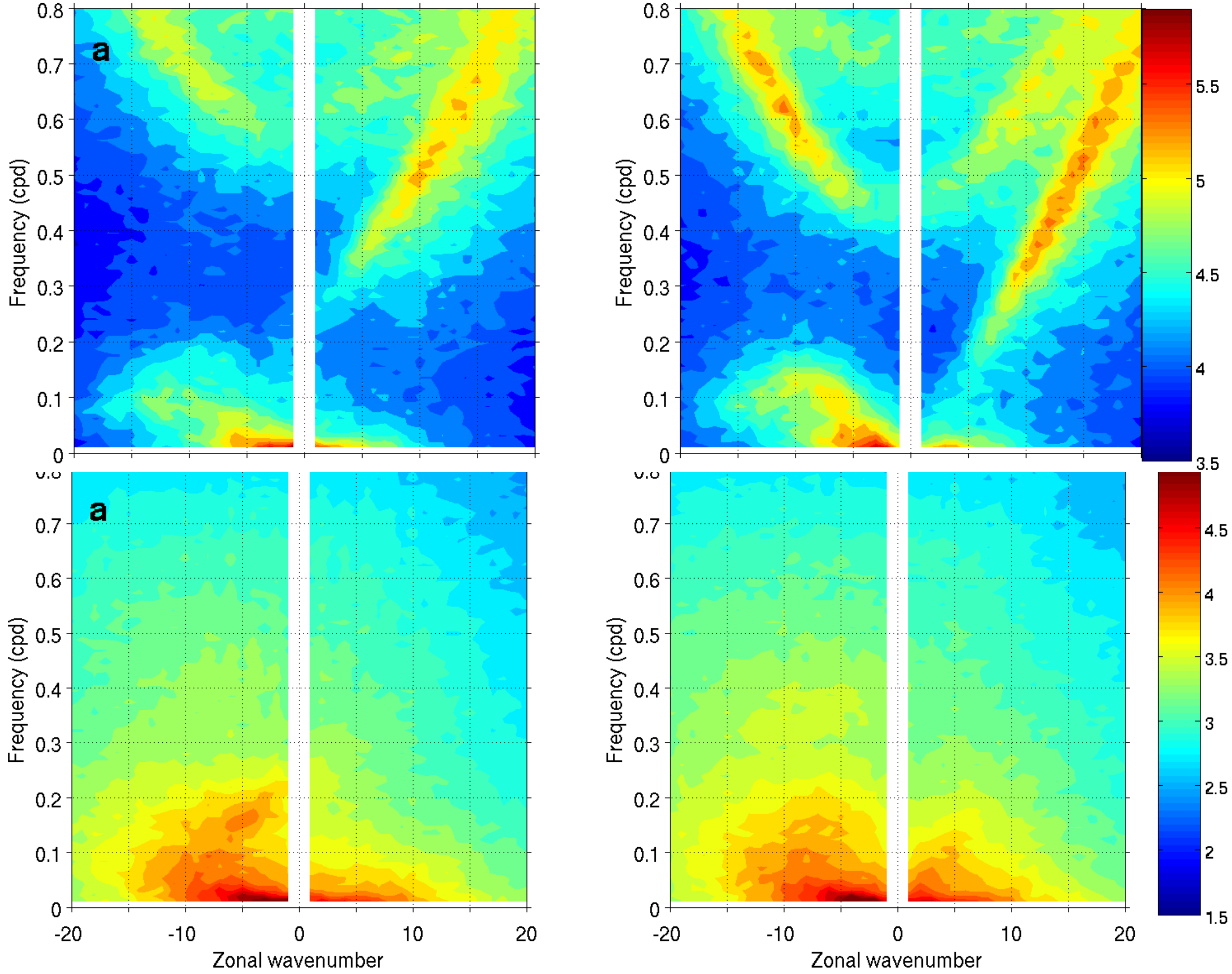


- The linear response function does not produce the same degree of self-aggregation seen with the CRM.
- Radiative feedback does enhance variance in column precipitable water.

With rotation: Spectra of 500hPa ω

Linear response function

Full CRM



Potential differences between the CRM and the linear response functions

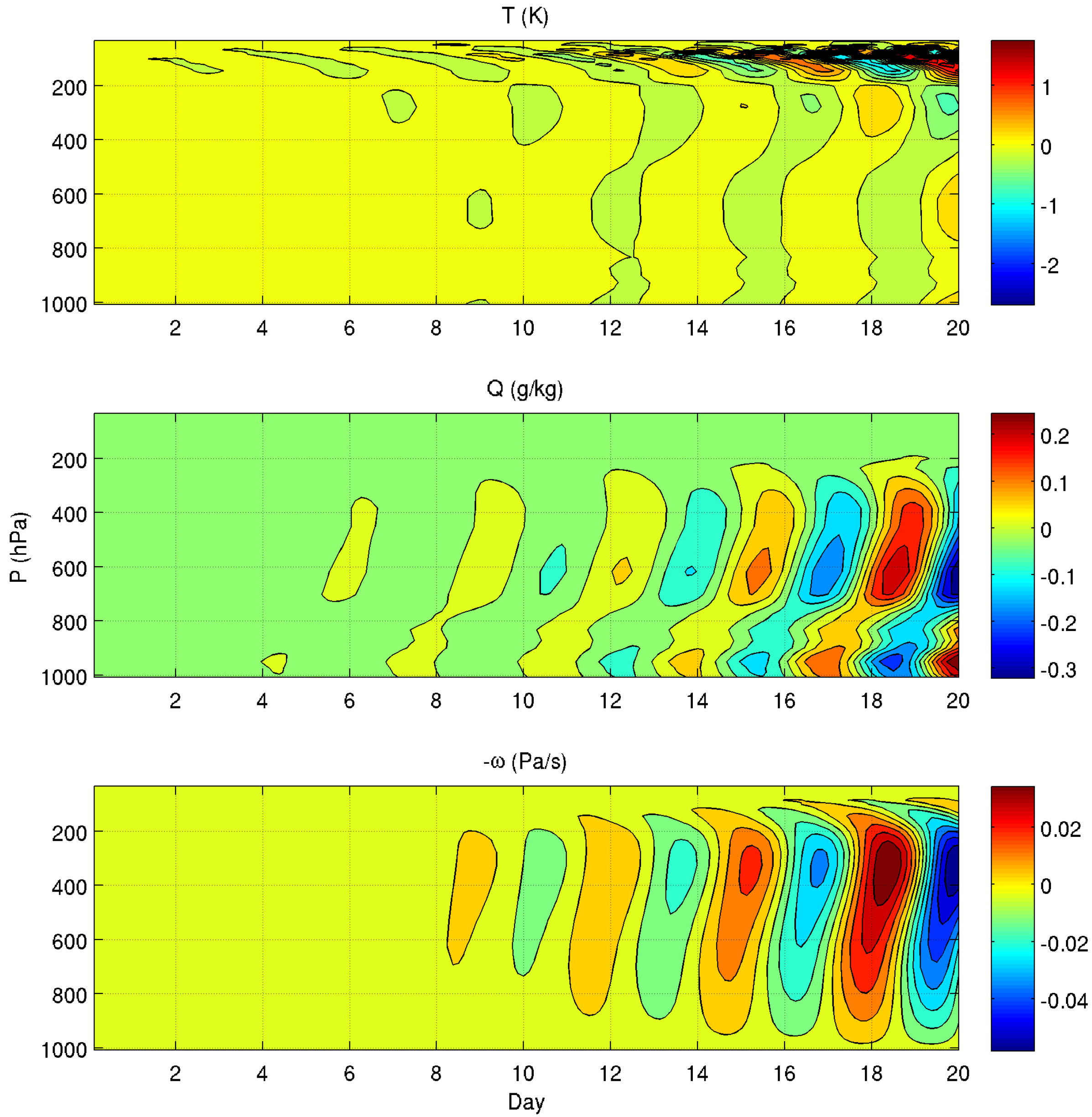
- Stochastic noises
- Time lag in the convective response
- Inaccuracies in the linear response functions
- Nonlinearity (state dependence) in the convective response

Coupling the linear response function to linear gravity waves

RCE reference state:
(5000km horizontal wavelength)

Convectively coupled waves grow

(Recall that these are linear calculations)

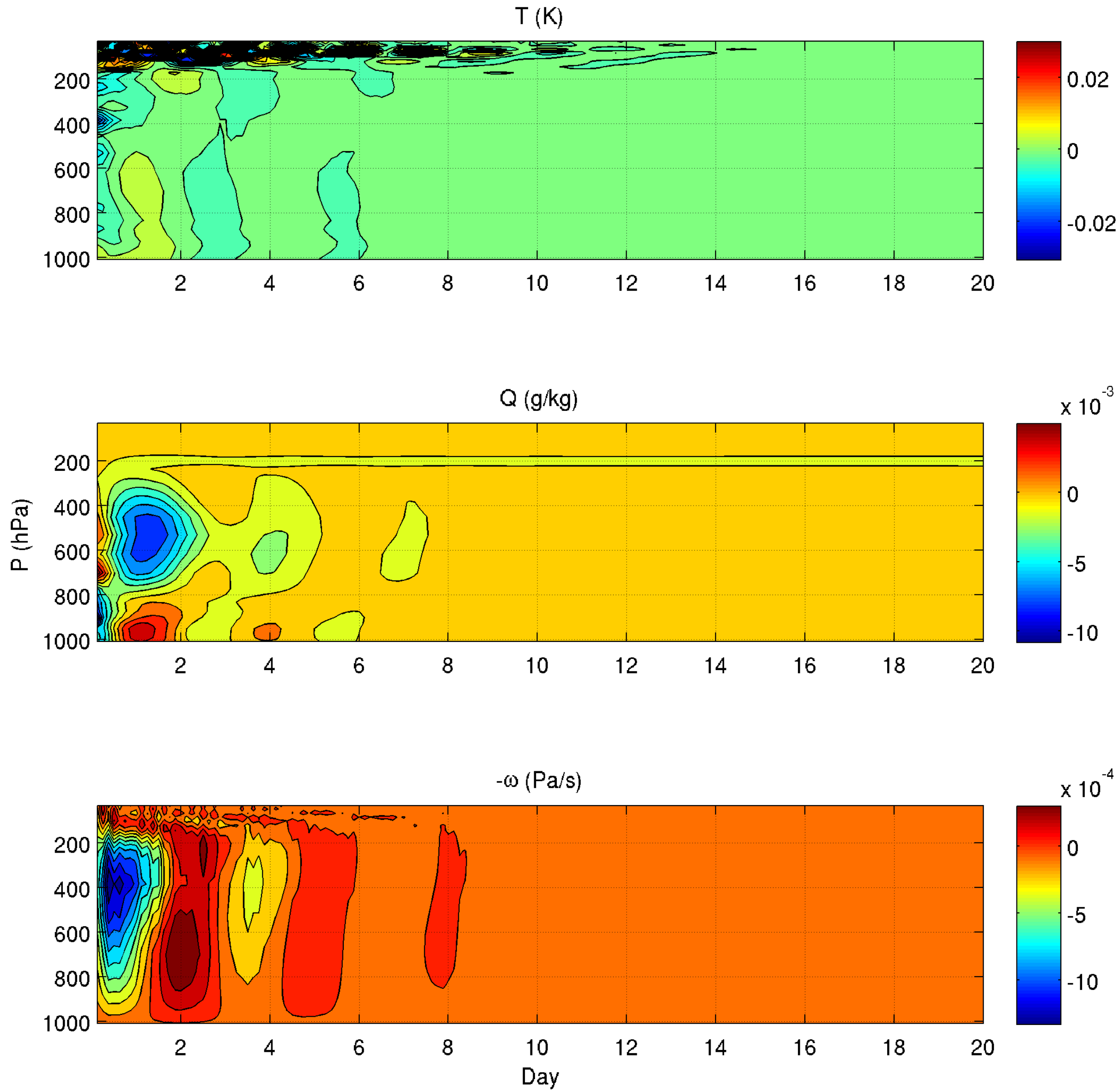


Coupling the linear response function to linear gravity waves

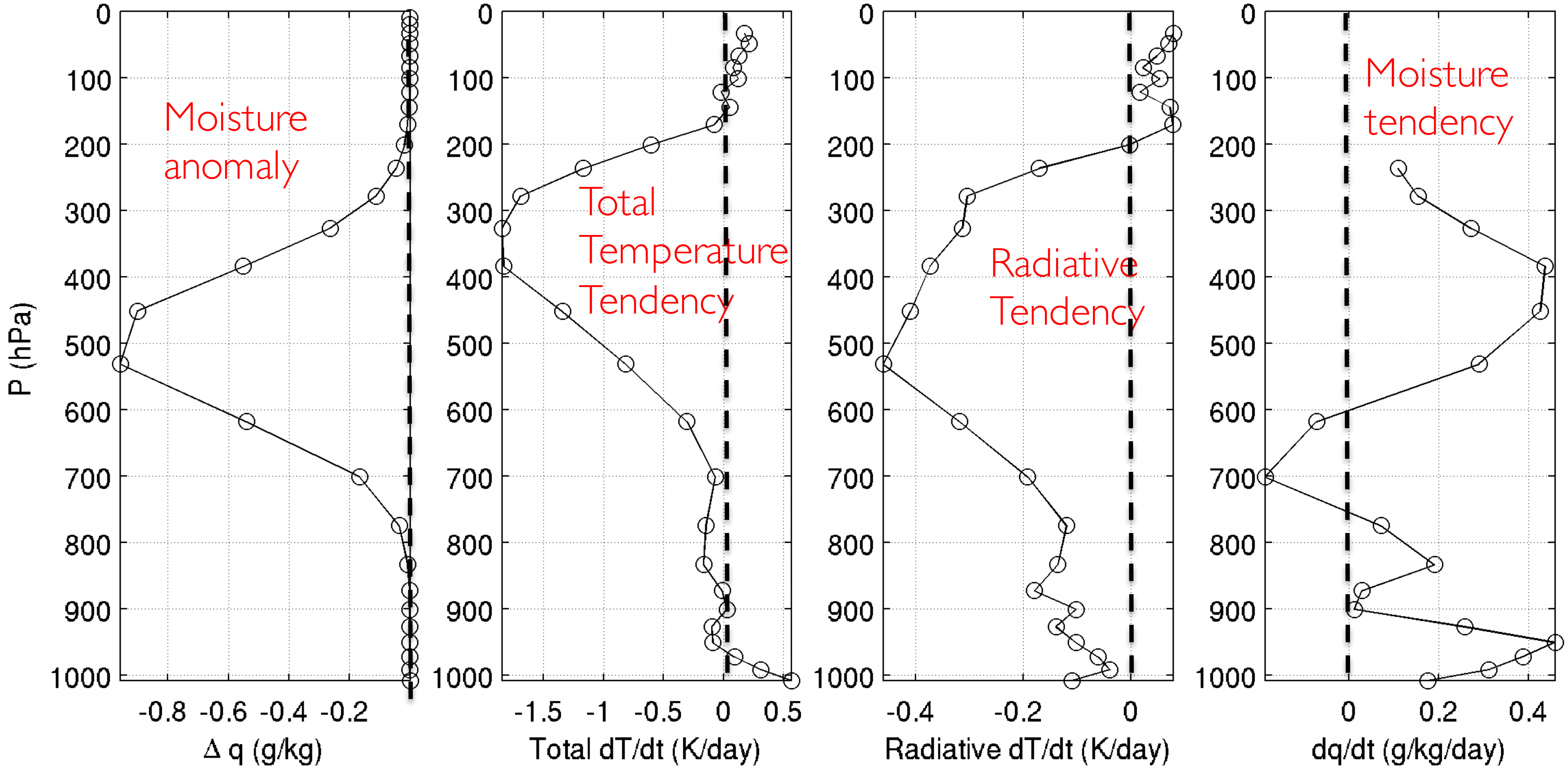
Moist reference state:
(5000km horizontal wavelength)

Convectively coupled waves decay

(Recall that these are linear calculations)

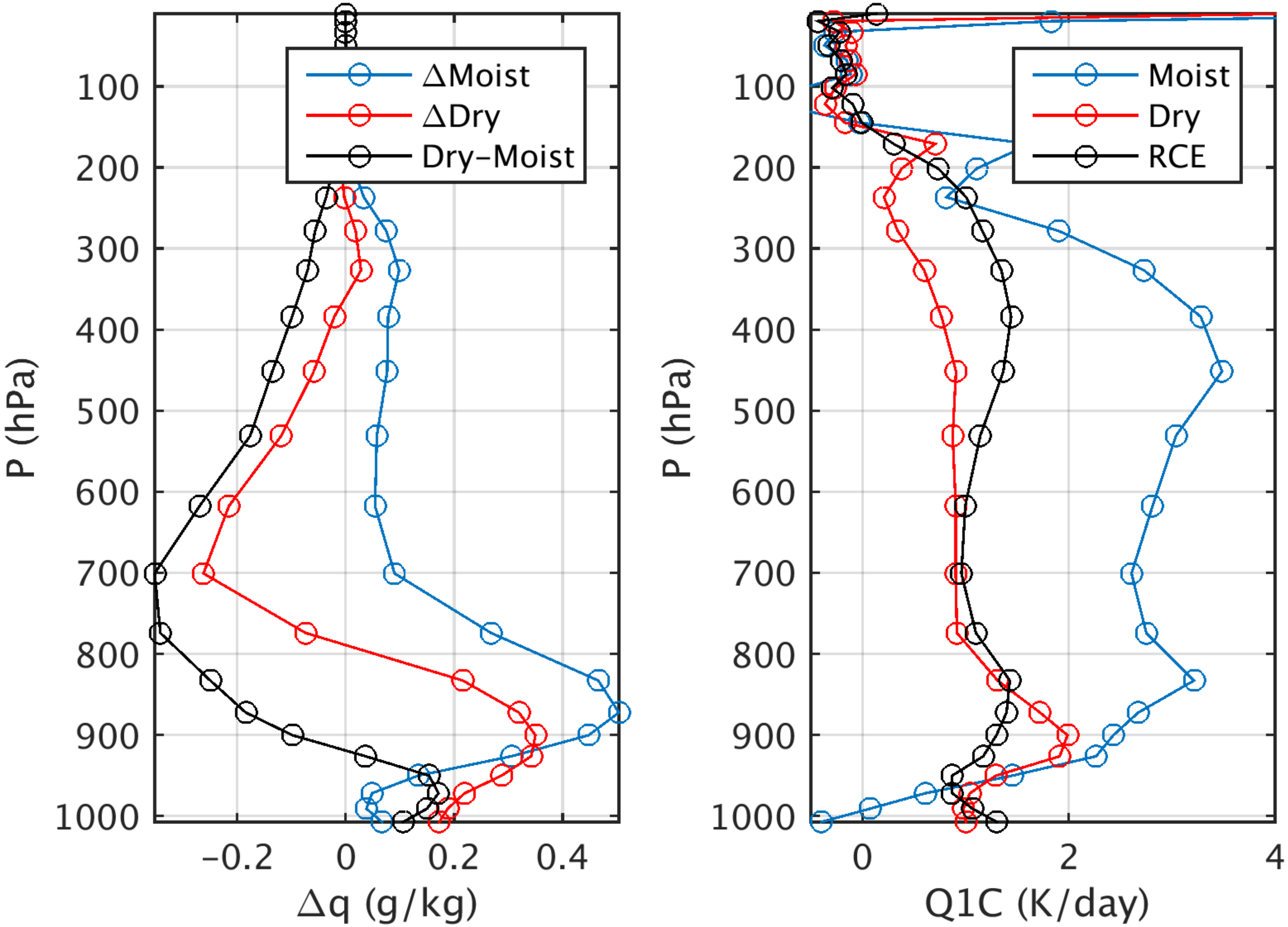


An illustrative example



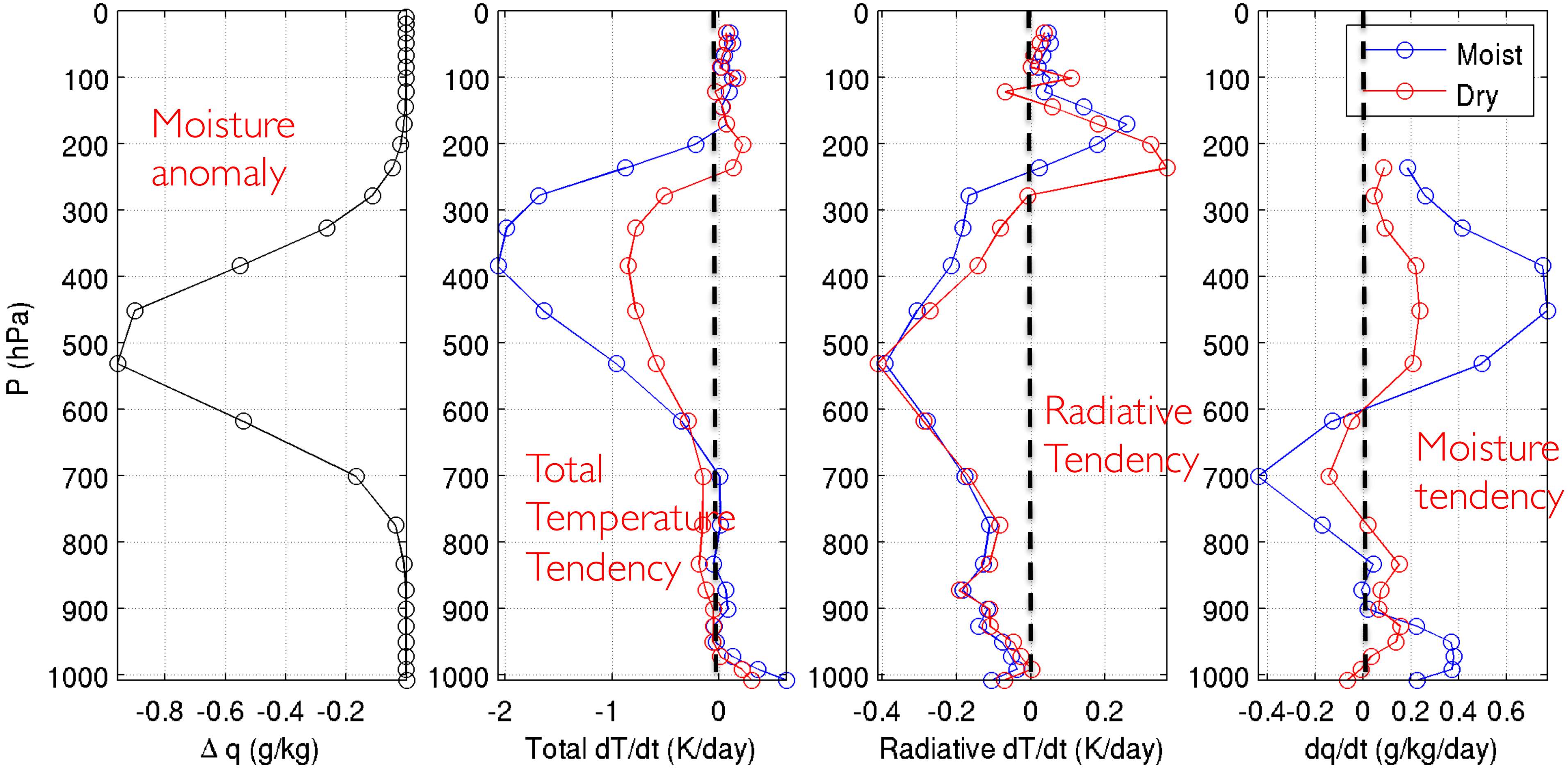
Radiative cooling can lead to amplification of the original dry anomaly, but convective moistening can also damp it.

A moist and a dry reference state



Mean precip: 8.0mm/day, 3.5mm/day, 2.8mm/day, respectively

Responses to a dry anomaly



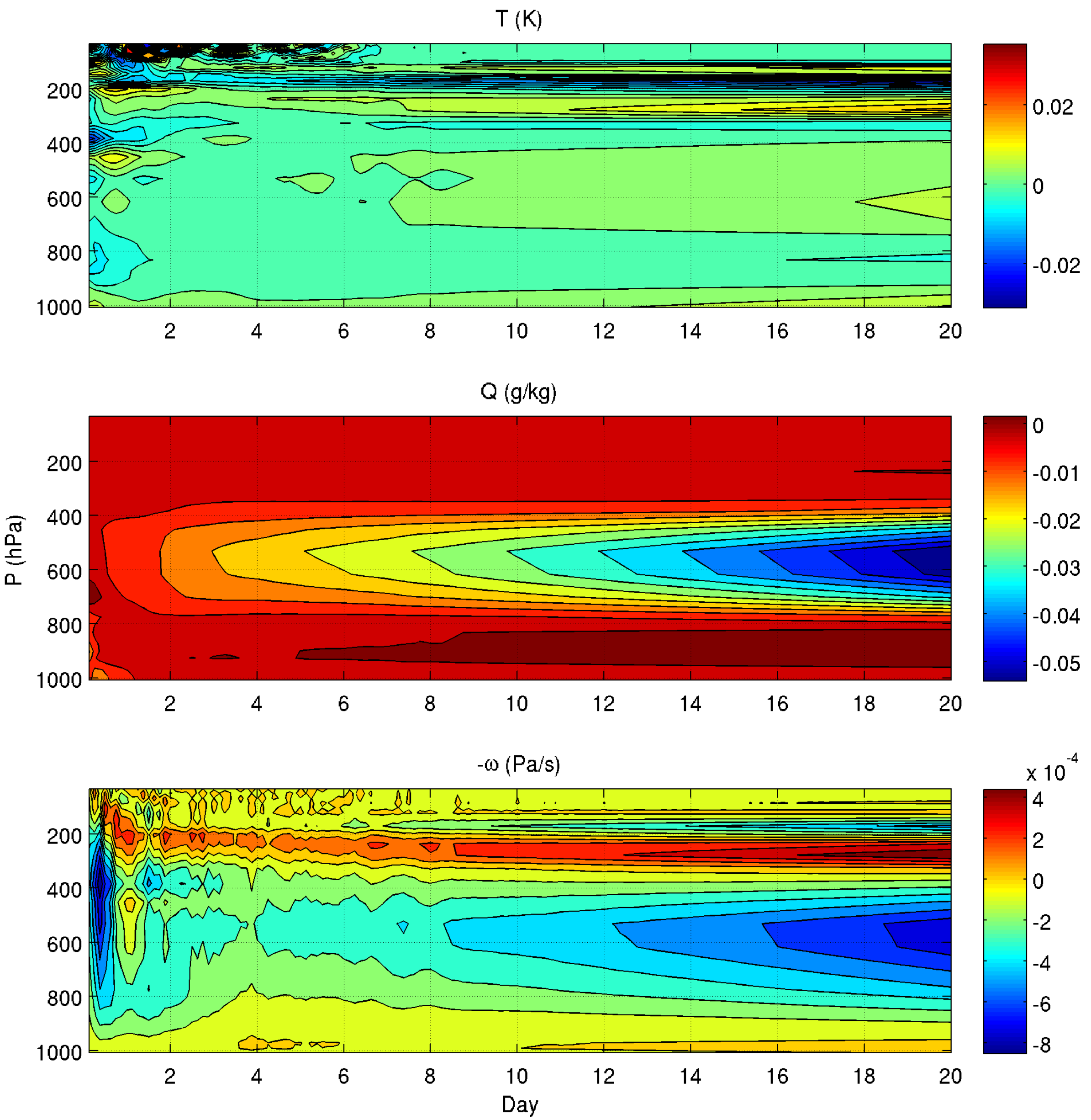
The dry and moist reference states have similar radiative feedbacks but the convective damping of the moisture anomaly is weaker in the dry state.

Coupling the linear response function to linear gravity waves

A dry reference state: (5000km horizontal wavelength)

A stationary "moisture" mode grows

(Recall that these are linear calculations)



- Linear response functions of a cloud resolving model support linear radiative convective instability hypothesized in Emanuel et al. (2014).
- However:
  - Cloud radiative feedback is important
  - The instability is stronger for a dry mean state, which could explain why growth of dry patches dominates the self-aggregation.